

NN31545.0491

INSTITUUT VOOR CULTUURTECHNIEK EN WATERHUISHOUDING

NOTA 491, d.d. 21 november 1968

Rules of transfer of water management experience,
with special reference to the assessment of drainage
design constants

W.C. Visser

BIBLIOTHEEK
STARKHOEDEN

Nota's van het Instituut zijn in principe interne communicatiemiddelen, dus geen officiële publikaties. Hun inhoud varieert sterk en kan zowel betrekking hebben op een eenvoudige weergave van cijferreeksen, als op een concluderende discussie van onderzoeksresultaten. In de meeste gevallen zullen de conclusies echter van voorlopige aard zijn omdat het onderzoek nog niet is afgesloten. Bepaalde nota's komen niet voor verspreiding buiten het Instituut in aanmerking.



1787266

Vol. 11, No. 1, January 1, 1917

THE JOURNAL OF THE AMERICAN MEDICAL ASSOCIATION
PUBLISHED WEEKLY
535 N. Dearborn Ave., Chicago, Ill., U.S.A.
Subscription Price: Five Dollars Per Annum in Advance

Vol. 11, No. 1

THE JOURNAL OF THE AMERICAN MEDICAL ASSOCIATION
PUBLISHED WEEKLY
535 N. Dearborn Ave., Chicago, Ill., U.S.A.
Subscription Price: Five Dollars Per Annum in Advance

Rules of transfer of water management experience,
with special reference to the assessment of drainage
design constants

W.C. Visser

Institute for Land and Water Management Research, Wageningen,
Netherlands.

Introduction

In the countries, where water management projects have already been in existence for a long time, a vast knowledge is available concerning the rules and constants governing the reaction of the growth and productivity of crops on soil and water conditions. This knowledge is partly founded on soil physical and plant physiological experiments and observations, another part rests on experience gained in the field. Properties of importance for the design but varying from field to field as permeability and moisture retaining capacity, generally will be measured. Properties with a regionally more constant nature as rainfall and evaporation, often will be dealt with along empiric lines.

Often the project engineer will have to make plans for areas where the validity of design constants derived from empirical experience is no longer ensured. This will not only occur within the Netherlands, where water management primarily was developed for arable clay soils and now is applied to other conditions as sandy soils, horticultural land or orchards. Here the earlier experience will not necessarily hold. The soils may differ considerably and more expensive crops will require more intensive and more costly solutions. Even more poignantly does the problem of transfer of knowledge and experience show up in projects in the developing countries, where soils and hydrology, climate, crops and economy are vastly differing from the situation in the country where the experience is gained. Here the project planner has to decide what part of his experience and knowledge still will hold.

The answer that all the research for these different conditions will have to be done all over again is not acceptable. It is of importance therefore to question what the rules of transfer of existing knowledge to other environments are and what can be done to shape the research in countries with advanced water management experience in such a way, that transfer to areas where this experience is lacking is possible. For such a transfer it will not only be necessary that the result is sufficiently accurate, but it also will have to satisfy requirements of simplicity and manageability. The last requirements limit to a large extent the freedom of evolving theories which allow an easy transfer.

General rules of transfer

The classic rule of transfer is, to reduce relations to accurate and generally accepted functions or approximations and to avoid systematic errors. As an instance may be mentioned the incompressibility of water and the linear Darcy-law on which the hydrology is based. Another rule is, that the constants should be reduced to absolute physical constants or should be based on direct measurements. This minimises the random errors.

In water management research, as in all comprehensive problems, the influence of the availability of time, money and specialists will have to be added to the usual requirements of the classic type of research. The gain from additional accuracy has to be set against the additional time that will be required or against the availability or the cost of a better expert to carry out the research. This may change the characteristic of selection of the method from those of scientific elegance to those of an acceptable increment in accuracy.

In general the scientifically more accurate formula will provide a result with a higher degree of reliability. But it is known, that two formulae of vastly different complexity may yield nearly the same results over the whole range of variations. Further, the most accurate determination will often render the highest certainty of a correct result. But it is also known that results may prove to be very insensitive to errors of part of the parameters, whilst the sensitivity for errors in other parameters may be high. In such cases, it will not be difficult to select the more efficient formula or to decide on the amount of work to be spent on the accuracy of determination of the parameter with a large or small effect. The more difficult decision is, to assess which complexities or difficulties in the method a designer is able and willing to deal with and what may be considered to be entirely the task of specialists or scientists. The wide range of problems, present in any comprehensive project, sets a limit to the intensity of the attention which can be given to each problem. Wide applicability of the knowledge to be transferred will require a measure of attention for each aspect which holds a proper relation to the economic importance of that aspect.

The expectation that it will be possible to use data from one area in designs in other areas, will not often come true. What can be transferred, however, is the functional relation between causes and effects. The constants in this relation have to be assessed in the area of the project. What further can be transferred is the best way to assess such constants. This concerns not only

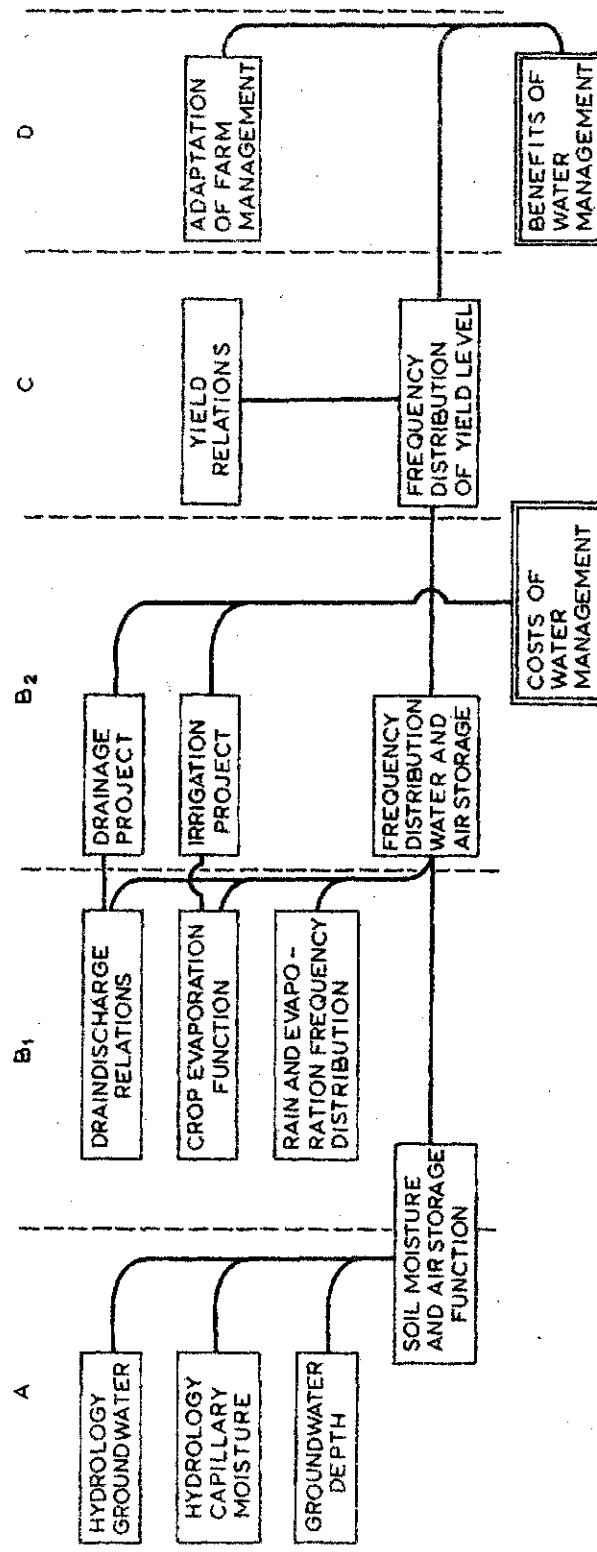


Fig. 1. Diagram of the aspects and interrelations of a water management project

the type of observations that give the best results with the smallest amount of specialist training, time and money. It also deals with the methods of adjustment, which make it possible to extract reasonably accurate values for the constants from observations of restricted accuracy. It seems advisable that closer attention should be given to the way in which observations are condensed to the desired information. Attention is also needed for the way in which conclusions as accurate as possible are drawn from inaccurate data. In the transfer of water management experience emphasis should not only be laid on technical and physical aspects, but also on the treatment of the observational data.

The diagram of relevant aspects

A water management project is based on four main aspects, depicted in fig. 1. The part, indicated with A, comprises soil hydrology. This deals with the properties governing groundwater flow and with the properties upon which the relations of soil moisture and soil air in the unsaturated zone are based. Part B deals with the depletion of the soil moisture stock by drainage and evaporation and its increase by rain or by application of additional water. Part B can be split up in B_1 , which describes the water balance, and B_2 describing the technical measures developed on account of the data from part B_1 . This part B_2 is today very much the main core of the water management design and derives its importance from the fact that the costs of the project are based on these informations. In part C the prognosis of the results on crop yield is listed, which is derived from the considerations in the parts A and B_1 . In Part D are indicated the effects of the change in the availability of water on farm management and the benefits that will be derived from the change in the water management situation due to the measures taken. Comparison of the costs in column B_2 with the benefits in column D make it possible to assess the efficiency of the proposed measures and to evaluate the measures as to their feasibility. The aim should be to determine with what detail and what margin of intensity the project should be carried out to ensure the optimum economic effect.

Comparing the course of investigations as depicted in this diagram with the usual practical approach, part C and D require the most active attention. These parts represent the most unpredictable uncertainties. The yield prognosis in part C, however, has the favourable property that very much is known about plant growth and that it is based on invariant plant physiological rules, applicable the world over. The influence of farm management in part D is far less well-known and - as it is based on human behaviour -

is not invariant. The adaptation of farm management to the moisture status of the field will in many areas only be known rather superficially and will often be the limiting factor for the accuracy of the expected results of water management design.

This part of the research, which is making steady progress, is generally based on the elaboration of information obtained with questionnaires. This technique is not applicable everywhere, however, so that this subject will not be discussed in this paper.

The rules of transfer for the prediction of crop response

The law of Darcy, basis of the worldwide application of the rules in hydrology, is also the basis - together with the formally identical law of diffusion - for the laws of plant response. This application of the linear laws of transport of matter on other growth factors than water is less commonly known and applied and will be discussed in detail. These crop - yield relations deserve attention because of the limiting position that yields have in comprehensive water management planning.

An important point is that a law of plant growth represents with a few additional relations an economic production function from which it is possible to derive the order of feasibility of improvement measures, or the marginal intensity of each separate improvement activity. In its original shape it shows how growth factors cooperate and how beneficial effects - for instance draining of excess of water - can be evaluated against the harmful effects due to moisture deficiency caused by such draining away of moisture.

The laws of the response of crops to water management measures are also valid for the solution of the normal problems of soil fertility management and fertilizer application and cannot without loss of accuracy be separated from this soil fertility aspect. Because the laws of plant growth constitute the area of contact between the technical and the agricultural specialist, this also requires a discussion in some detail.

General aspects of plant response

Usually it is assumed that the plant reacts on the depth of the groundwater-table. What the biological effect of this groundwater table might be - in the soil itself at this level no other effect than that of a water tension equal to the atmospheric pressure is present - has never been stated and a direct effect does not seem to exist. The plant reacts on the moisture stress and the moisture content in the root zone and on the rate of exchange of oxygen and carbon-dioxyde between the root zone and the atmosphere. Because there is a fixed

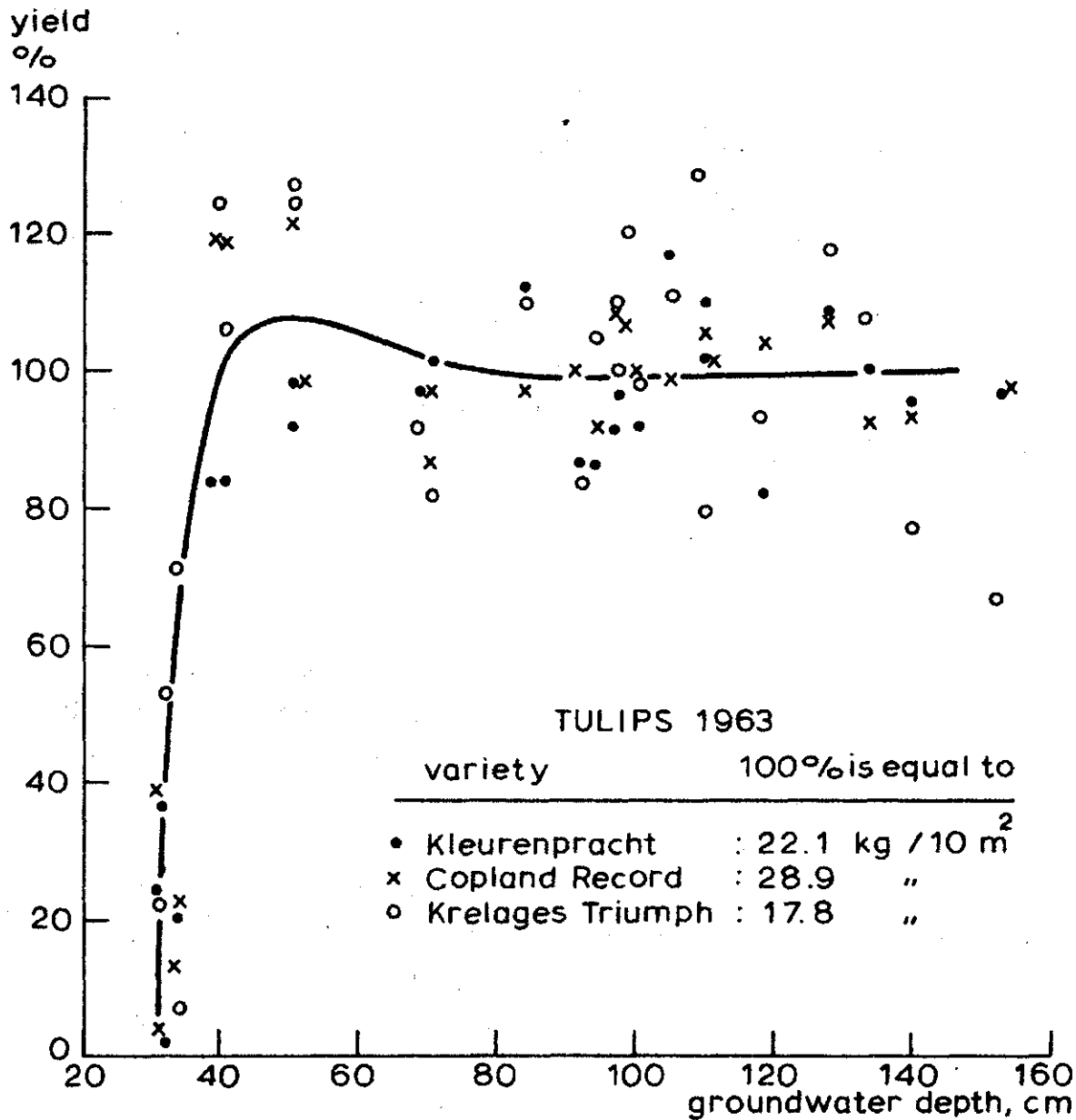


Fig. 2. Crop growth, depending more on air diffusion than on groundwater depth, reaches its optimum at low values of the depth of drainage if water drains readily from wide pores at low moisture stresses

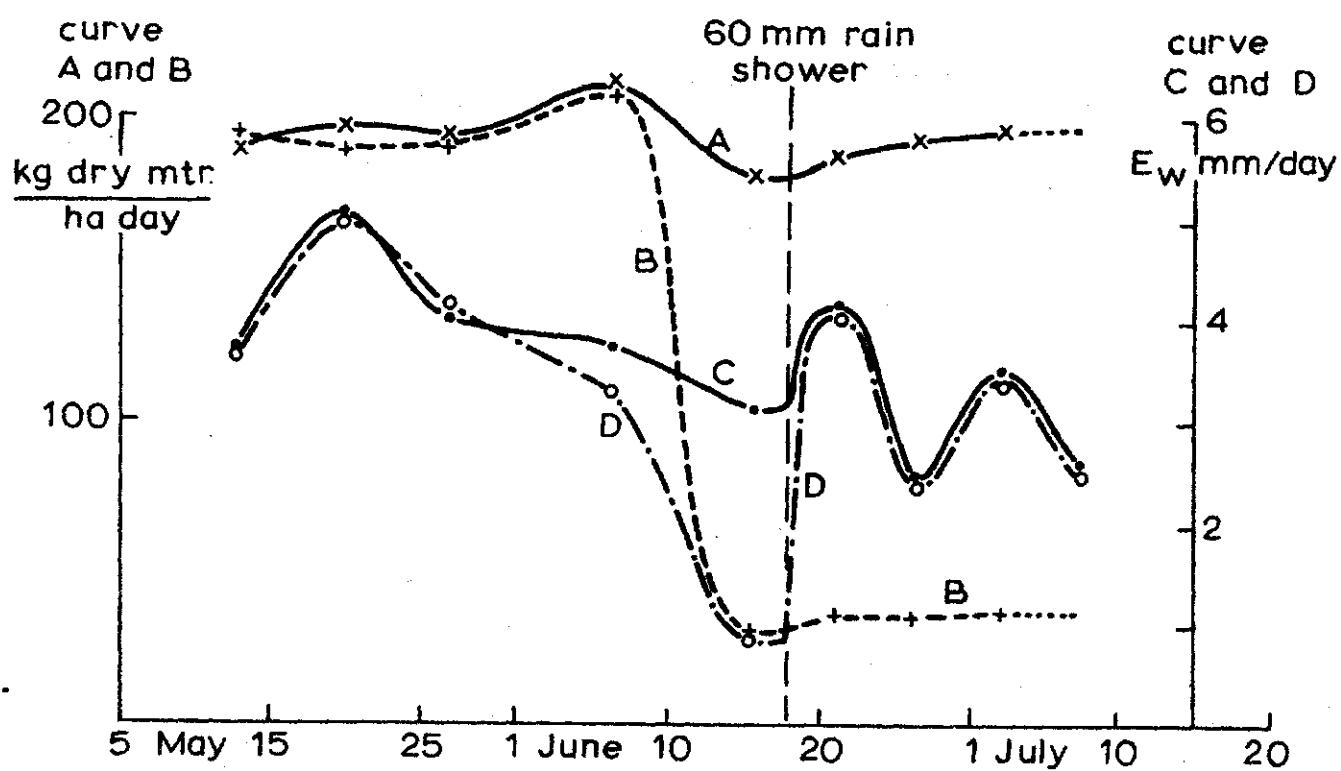
relation between the moisture content and the moisture stress, as well as between the air content and the air exchange coefficient for each separate soil and soil structure condition, the momentary reaction of the crop on water management measures can be largely explained by the moisture and air content of the root zone.

The plant, however, is not a constant entity in this relation, but can adapt itself to gradually changing situations. If the groundwater table draws down gradually, the roots follow the water table and the rate of growth will not be affected much. The more or less quick variations of the moisture conditions, superimposed on this slow lowering of the water table, have more influence. Experiment fields with a constant groundwater depth generally show optimal growth at very shallow, but constant, water tables which would cause grave trouble if used in practical design.

In fig. 2 the result of an experiment on the groundwater level is shown, where the constant depth of the groundwater table allows tulips to give an optimal growth at a very shallow groundwater depth*. Such a result shows what problems the design engineer will be up to, if he relies on scientifically conducted experiments with constant water depth and would construct in his project area a drainage system ensuring an optimum water depth as obtained on his experiment field. The first rainshower would convince him, that a field experiment without the variations in the groundwater table which normally occur, is not a good guide in water management design. These sudden changes vary from drought damage due to dessication, to moisture excess due to inundation. Whilst the slow variations in moisture relations are sufficiently characterized by the water depth in summer and winter, a description of the variation in moisture condition of short duration is required in addition. This is done by recording the moment and length of moisture excess or deficiency.

Next to a distinction in slow and rapid variations, another distinction has to be made in reversible and irreversible damage. A small degree of shortage or excess of water will lower the rate of growth, but as soon as the unfavourable situation is lifted, the plant grows on at the original rate. The reduction of the growth rate is reversible. If, however, the unfavourable situation is of a high intensity, impairing the production mechanism of the

* Valk, G.G.M. van der and J.A. Schoneveld: De reactie van tulpen op grondwaterdiepte en profielopbouw. Meded. Dir. Tuinbouw 1964, 27, 12; 631-639.



67C 8 1/3.4

Fig. 3. Heavy irreversible damage was done to wheat on non-sprinkled plots (curve B and D) by a dry week, reducing the daily increase in yield, which after restoring the soil moisture conditions, does not - in contrast with evaporation - return to its previous level

plant, improvement of the unfavourable situation is at best followed by a slow recovery of the growth rate. If the production mechanism can still be restored, the damage is slowly reversible, but otherwise irreversible with the death of the plant as ultimate limit.

In fig. 3 an instance of this irreversible reaction is given. For a sprinkled and a non-sprinkled object the daily increase in dry weight is given by the curves A, respectively B. The curves C and D show the daily evaporation. The rainy month of May shows, that dry matter increase and rate of evaporation on the two objects are the same. June was very dry up to 18th and on the non-sprinkled field the evaporation rate and the daily dry matter production decreased rapidly. In the last 5 days the moisture tension in the non-sprinkled soil dropped till pF 4.5. Then a rainshower of 60 mm restored the soil moisture status. Line D for the non-sprinkled field rises to the level of the sprinkled field and the following month, due to repeated rainfall, the two experiment plots show an equal evaporation. The dry matter production of the non-sprinkled field, however, does not return to its original level, as line B shows. In the dry week the production mechanism was apparently severely damaged and in the following three weeks of the experiment this damage was not repaired, notwithstanding that the crop still lived and evaporation went on at a normal rate. A small amount of water between 12 and 18 June presumably could have prevented the irreversible damage and would have given a very satisfying ratio between amount of water given and yield increase obtained.

The distinction between steady versus unsteady water relations and the distinction between reversible and irreversible damage is presumably of more decisive importance for project design than up to now has been thought. But although the unsteady and the irreversible effects are of more importance for crop growth than the steady and the reversible effects, the latter effects are much easier to counteract. The primary aim of project design should, however, be to prevent the ill effects of unsteady conditions and the irreversible effects. In this respect it should always be remembered that field experiments on moisture relations and crop yield often deal with steady effects which are of restricted value in project design. The better the moisture conditions were kept constant in the experiment, the more the results should be used with reserve.

A plant physiological basis of water management

The law of plant response can be based on three fundamental relations. The growth promoting factors are taken up by the plant via a linear process

of transportation, which may be the Darcy law for moisture flow or the law of Fick for diffusion*. The transportation velocity is given by:

$$v = \frac{x_2 - x_1}{R_{12}}$$

The flow resistance R tends to be constant. The rate of growth q is directly related to the transportation velocity v and the factor c , describing the ratio of growth rate q to uptake rate v also is nearly a constant.

The following relations hold:

$$q = v/c \qquad q = \frac{x_2 - x_1}{cR_{12}}$$

$$\frac{1}{c} R_{12} = a \qquad a = \frac{q}{x_2 - x_1} \approx 0$$

The value of a , equal to the inverse of the product of two magnitudes which tend to be constant, is nearly equal to the ratio of the growth rate divided by the gradient.

That the inverse of the ratio between x and q has to be taken, meaning that it is not a resistance but a conductivity which tends to be constant, has to be understood as follows. The supply of the growth factor $a(x_2 - x_1)$ is the independent factor, the growth rate q is the dependent factor. The plant reacts according to a ratio of the yield per unit value of the nutrient supply. Therefore the reaction is governed by the magnitude of $q/(x_2 - x_1)$ and not - as might be assumed as alternative - by $(x_2 - x_1)/q$.

The difference $a = q/(x_2 - x_1)$ can be given for any growth factor to be inserted in the yield equation. The significance of such a difference is, that it represents a stress, which the plant has to overcome in taking up a satisfactory amount of each separate growth factor. The larger the difference is, the larger this stress becomes. Now it is assumed that the sum of the deviations $a = q/(x_2 - x_1)$ - the size of which indicates the degree of intensity of the struggle for homogeneous growth - if expressed in a relative scale to account for differences in magnitude, will be zero. This supposition means that this struggle leads to an equilibrium situation with respect of the combined stresses. This equilibrium defines the magnitude of the excesses and deficiencies, which govern the ultimate growth rate q .

* Visser, W.C.: Ein Ertragsmodell für günstige und schädliche Umweltfaktoren. *Studia Biophysica* 1968. In press.

Visser, W.C.: Anwendung der parametrische Biologie auf praktische Probleme. *Studia Biophysica* 1968. In press.

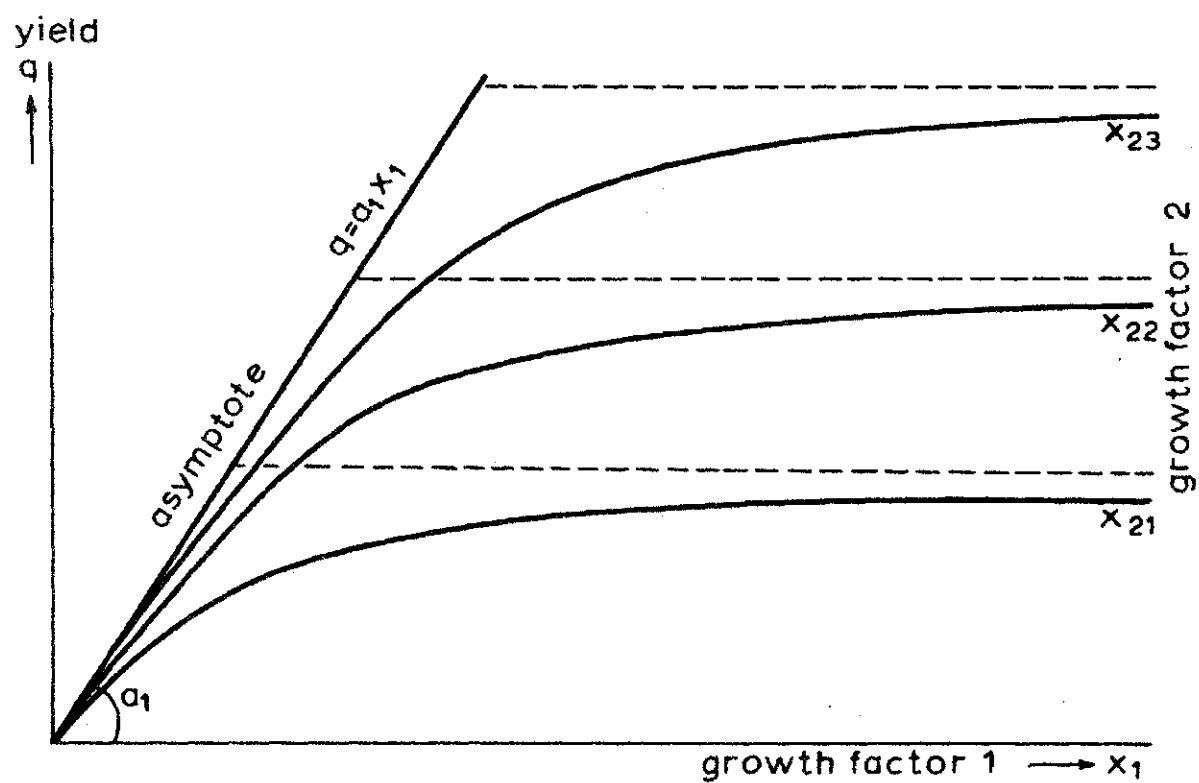


Fig. 4. Graphical representation of the yield function, identical to the representation of the law of the limiting factors or the growth model according to Blackman

This minimum stress is defined by:

$$\sum_{i=1}^n \frac{d\left(a_i - \frac{q}{x_{i2} - x_{i1}}\right)}{\left(a_i - \frac{q}{x_{i2} - x_{i1}}\right)} = 0$$

Integration provides an equation in which the maximum of the growth rate or of the uptake capacity due to the genetic properties, can be inserted by assuming that the uptake of nutrients, leading to maximum growth Q , is proportional to the value of this Q . So a growth equation can be obtained reading:

$$\left(1 - \frac{q}{Q}\right) \left(1 - \frac{q}{a_1 x_1}\right) \left(1 - \frac{q}{a_2 x_2}\right) \dots = F_1 \quad (1)$$

The symbol F stands for the integration constant and has the meaning of a constant, depicting the flexibility with which the plant can adapt itself to variation in the availability of the growth factors x_1, x_2, \dots . The formula should for theoretical reasons contain all growth factors, but for practical application the most important ones will do. The factors which cause damage to the growth mechanism of which the magnitude is represented by Q , should be inserted in the formula as some function reducing the value of Q .

The formula describes a growth rate or a dry matter increase per unit time. Field experiments generally work with yields over the full growth period and to use these data it is usually assumed that the factors a_i are constant over this period. Is this true, then the equation can be used as a yield function.

Assessment of the constants

Graphically represented, the law of plant growth takes the shape as depicted by fig. 4. The curves approach an oblique and a horizontal asymptote. This horizontal asymptote can be situated at different levels of q . In the simplest case the oblique asymptotes for increasing values of other factors coincide. The equation has the useful property, that the shape of the curves is practically identical. They only differ due to a translation parallel to the direction of the oblique asymptote.

In fig. 5 upper half, the curves for the response of the crop to nitrogen, obtained from field experiments, are given as an instance. If these curves are shifted correctly, the observations become grouped along one single line, which satisfies the equation.

$$\left(a - \frac{q}{x}\right) \left(1 - \frac{q}{Q}\right) = F_1$$

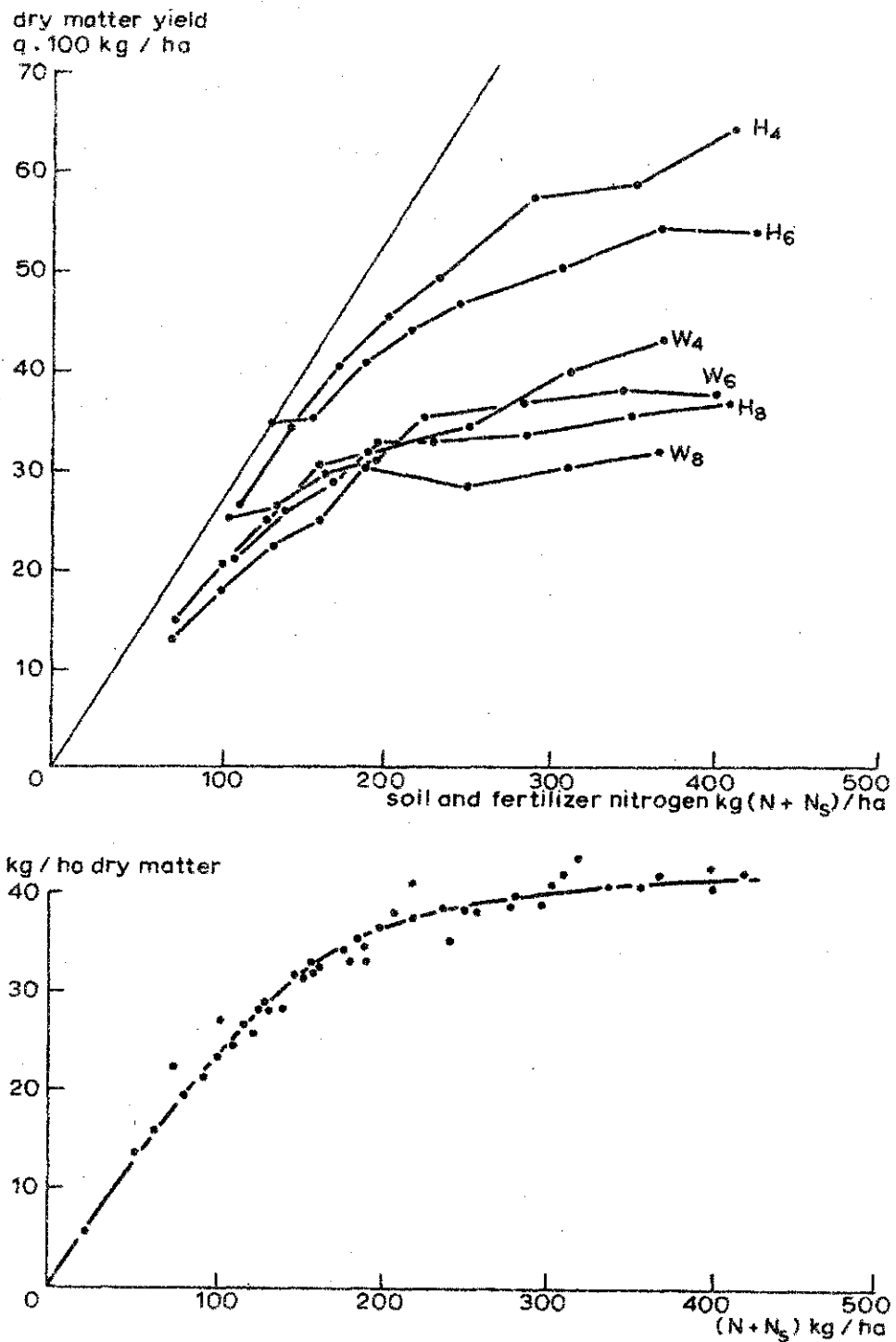


Fig. 5. With a good approximation, yield curves can be brought to coincidence by shifting along the oblique asymptote, or in case of differences in the growth factor from unknown sources, by an additional horizontal shift of which the lower graph presents the result

The level of the horizontal asymptote represents the value of Q . The inclination of the oblique asymptote represents the constant a . A few trials provide the value of F .

The project researcher generally will not have field experiments on soil moisture conditions and crop response available. He can, however, select in the field a number of spots with large differences in moisture relations. By collecting yields every week and determining the moisture conditions - ground-water depth or soil moisture content - at the same moment, data are available to solve the problem. For sufficiently narrow intervals of growth factor x_1 , the yields q are plotted against x_2 , and for intervals of x_2 the q is plotted against x_1 . On the scatter diagrams of this type, the translation technique is applied and the constants can be found.

This elaboration makes it also possible to treat the problem when q depends on more than one growth factor x . Often the investigations will be sufficiently accurate if only one growth factor is taken into account. In fig. 6 an instance is given of the influence of the volume percentage of air on the growth of grass on peat soils.* This growth was determined during four successive cuts and the separate yields were plotted against the average air content observed during the growth interval. Because it is known, that moisture conditions do not influence growth according a linear, but according an exponential relation, the logarithms of yield and air content are plotted against each other.

The small graph, given as an annex, is shown as a proof that repeated yields give a better result than total yields. In the annex the sum of the four cuts is plotted against the average air content over the entire growth period. This integration levels off the variation in the resulting scatter diagram and by decreasing the range of scatter gives far less convincing results. If one takes moreover into consideration that the growth relation is in reality an equation for the rate of growth and not for the total quantity of growth, the theory and the practical results both point in the direction of taking repeated yield determinations.

A physical basis of water management

The management of the groundwater depth or the moisture content in the root zone aims at obtaining the optimum relation between crop yield and several other magnitudes. These may be the amount of water or the discharge capacities available or the costs of water management practices.

* Titze, E.: Der Wasser und Lufthaushalt der oberen vererdeten Schicht des Klenzer Niedermoors und sein Einfluss auf den Ertrag.
Inaugural Dissertation Rostock, März 1966.

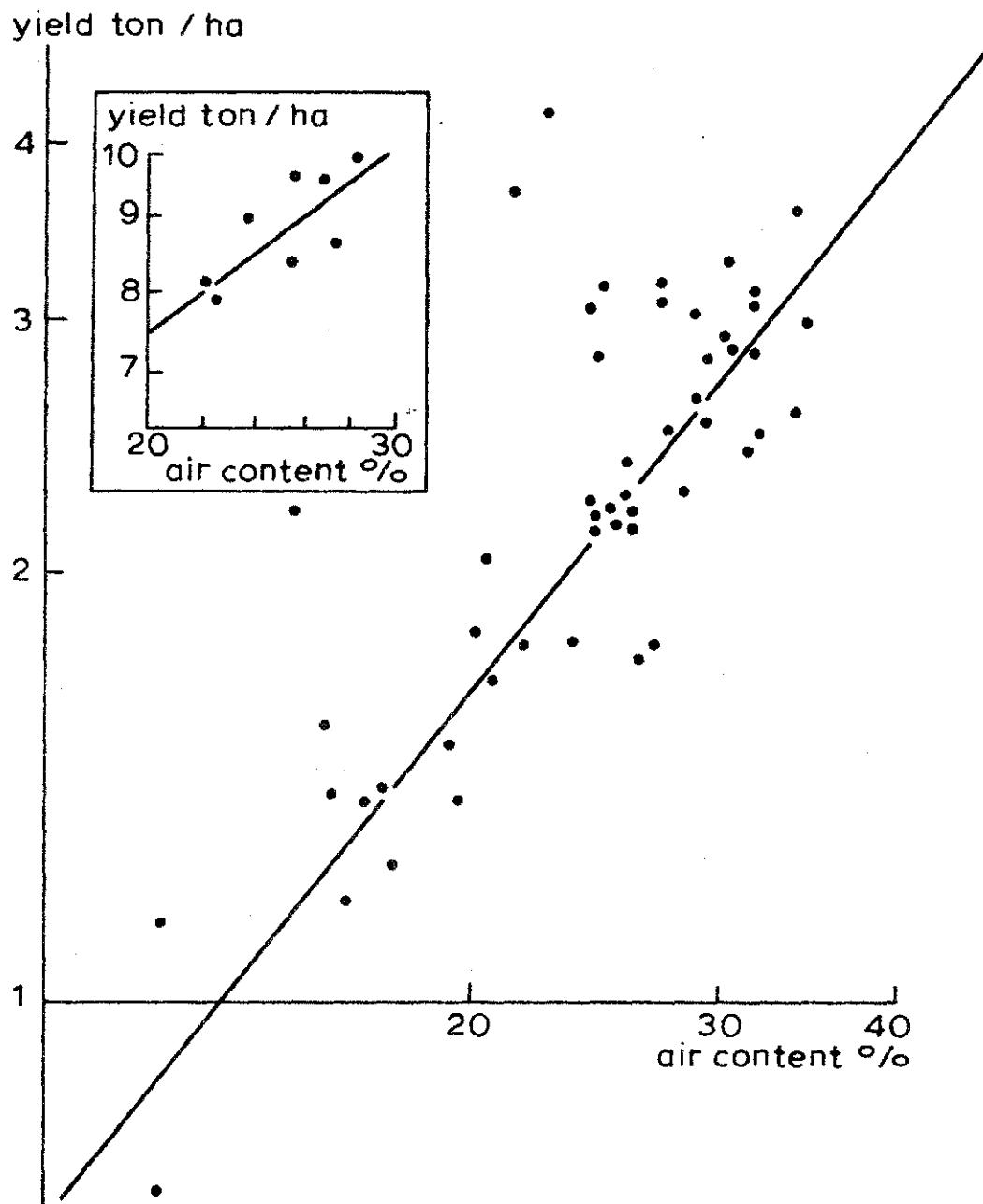


Fig. 6. Repeated harvests, plotted against the level of the growth factor in the time interval of growth, presents deeper insight into the growth relations than the total growth against the average level of the growth factor, as presented in the inserted graph

The water management conditions which should be attained, depend for their main aspects on four relations, rendered by the following formulae.

$$\text{Discharge} \quad \left\{ 1 - \frac{D}{\frac{8k_1 d_1}{L_1^2} (z - S_1)} \right\} \left\{ 1 - \frac{D}{\frac{8k_2 d_2}{L_2^2} (z - S_2)} \right\} = F_2 \quad (2)$$

$$\text{Evaporation}^* \quad \left\{ g - \frac{E_r}{E_o} \right\} \left\{ A - \frac{E_r}{(v' - v_{wp})} \right\} = F_3 \quad (3)$$

$$\text{Soil moisture profile} \quad (1 - e^{-\alpha z_w}) \left(1 + \frac{z_o - \frac{1}{\alpha} \frac{v_c}{k_o}}{z_o + z_s} \right) = (1 - e^{-\alpha \psi}) - \frac{1}{z_o + z_s} \frac{v_c}{k_o} z_w \quad (4)$$

$$\text{Desorption curve} \quad \psi = \frac{G(P^* - v)^n}{v^m} \quad (5)$$

Equation (2) relates the discharge D to the pressure head existing between the groundwater level z_g and in succession the water level in a nearby shallow ditch S_1 and a more distant and deeper main water course S_2 . All these groundwater levels are expressed as depth below soil surface. The insertion of two drainage bases in the formula is in accordance with the practical experience that generally two discharge effects can be observed, but a third level, if exerting an effect, has too small a drainage capacity to allow determination. Third effects are therefore neglected here. However, limiting the discharge model to only one drainage basis would neglect effects of too large an importance. The relation between discharge and gradient is assumed to be linear. For drainage with deep impervious layers and large drain distances this often will hold good. In case of tile drainage a term of the second power of z_g can be added.

Equation (3) states that the evaporation rate depends on alternatively the evaporative capacity of the atmosphere E_o - a value which can be calculated, but often can more reliably be determined as pan evaporation - or on the

* The word evaporation is used as a general description for the transition of liquid to vapour without any distinction of the process or origin of the evaporated moisture.

moisture content of the root zone v^{**} . The zero evaporation appears at a low moisture content v_{wp} known as wilting point.

Formula (4) relates the height above groundwater z_w and the soil moisture stress ψ at that point, with the velocity of capillary flow v_c . In this relation a parameter z_0 is inserted, which indicates the level at which the soil moisture is extracted.

Formula (5) shows the mathematical representation of the desorption curve, which relates the soil moisture stress to the soil moisture content.

The four formulae together account for the discharge, the amount of moisture in the profile and the evaporation, which, together according to the water balance equation, add up to the rainfall. Further represented are the water level in the soil, in ditches of a first and second order and the moisture content in the root zone. Problems on drainage or on water application can be solved, but also the discharge data, integrated for a catchment area, may be used as a basis for river regulation designs. Because further the crop reaction depends on the air- and water content of the soil, the equation for the desorption curve represents the link between the hydrology of the soil and the plant response with respect to this hydrological condition. The four formulae together allow to calculate a large number of cross relations between important variables. The main problem, however, is what values the many constants should be given. The determination of the constants will be the problem to be discussed next.

Determination of the soil moisture parameters

Two methods are available with which even in conditions under which no laboratory is at hand or specialist assistance is available, the water management situation can be analysed. These methods are based on soil moisture determination or on groundwater depth readings. The determinations of the soil moisture content represent a valuable tool in studies on fields, where the groundwater table is deep, whilst the determination of the groundwater depth is most useful in studies on more shallowly drained fields.

The determination of the soil moisture content, to be expressed as a volume percentage, requires soil augers with which cores of a known volume can be taken and asks for drying and weighing. This can be done cheaply and

^{**} Rijtema, P.E., 1965. An analyses of actual evapotranspiration
Agric. Res. Rep. 659, p. 1-107; Techn. Bull. 32, Institute for
Land and Water Management Research, Wageningen.

simply, as described by Bourrier^{*}. A roman balance can be constructed at very low cost, the drying can be done by mixing the soil with common household spirits which, when lighted, evaporates the soil moisture.

A further valuable information is comprised in the desorption curve. Although this curve is not decidedly necessary, its availability can be helpful. At lower values of the moisture stress up to pF 2.7 the determination can be done with the sand tank method as described by Stakman.^{**} At higher stresses the extraction of water by sunflowers provides a simple and practical method to determine the wilting point at pF 4.2. The point for pF 6.0 can be obtained by determining the soil moisture content of a soil which, put in a dish, is left to reach equilibrium with the atmosphere in the laboratory. At least 5 points of the desorption curve should be assessed to obtain it with reasonable accuracy.

The soil samples for soil moisture content determination should be taken once a week in some four successive layers of 20 cm thickness during a sufficiently long period to ensure that the moisture relations are known under different conditions of rain or drought, heat or cold, cropped or bare soil. The accuracy of the moisture determination should be tested by comparing the observations of successive layers or weeks of sampling.

The groundwater depth readings should be made in a tube, constructed of clay drain tiles, plastic drain pipes or perforated metal pipe. The observations have to be done on a weekly basis. Together with the observation of the groundwater depth, the level of open water should also be read in the nearest water course to which the groundwater drains. If no water course is present and the groundwater flow is determined by a gradient in the terrain, the direction of this gradient has to be determined by levelling the groundwater depth at a number of points. The gradient ~~is~~ in this case has to be determined as the difference between the water levels at two points situated on the line of maximum groundwater slope. The observation wells should reach to a depth in excess of the deepest groundwater level to be expected, and should be repeatedly checked on silting up or decrease of the depth of the bottom of the tube.

* Bourrier, J.: Bulletin Technique de Génie Rural. Nr. 61, mars 1963, 64pp.

** Stakman, W. P. and G. G. v. d. Harst: Soil moisture retention curves I
Range pF 30-42

Harst, G. G. v. d. and W. P. Stakman: Soil moisture retention curves II
Range pF 0-27

Nota 159 and 81, Inst. for Land and Water Management Research.

The water balance derived from moisture contents

The soil moisture contents are first plotted for the successive layers against each other - as given in fig. 7 - or against the total moisture content in the part of the profile that was sampled. These curves provide a first indication of the moisture content at field capacity and wilting point for the upper layers, in fig. 7 of the order of 27 and 7.5%. Considerations of the following type should be taken: in the linear right hand part of the line the moisture contents of the two layers vary in relation. At a moisture content of 17% the data diverge due to evaporation in the top layers. Would this evaporation not occur, then the moisture content could only lower due to drainage. And a lowest moisture content of the lower layer of 5.5% would match with a lowest moisture content in the upper layer of 14% and represent an approximate equilibrium situation, depending on the depth of the water table of some 10 m. This would be therefore the moisture content at 10 m moisture tension or pF 3.0. This example may show that moisture contents of successive layers are able to convey a first impression as to the soil moisture retaining properties.

The moisture content data are used to determine the terms of the water balance in a way as indicated in the following example. The moisture contents of the layers are added and the sum represents the moisture volume v_T of the profile over the full depth of sampling. Also the average moisture content v_r of the main root zone is calculated. The following table is then constructed:

		from:
Moisture volume v_T , 5 May	143.0	analyses
Moisture volume v_T , 16 May	<u>126.4</u>	analyses
Stored moisture	16.6	calculated
Rainfall 5-16 May	28.3	measurement
Sprinkling irrigation	<u>0.0</u>	application
	44.9	calculated
Moisture content root zone v_r	28.1	analyses
Pan evaporation	36.6	measurement
Multiplication factor $n = E_r/E_p$	<u>1.15</u>	calculated
Real evaporation $E_r = nE_p$	<u>42.1</u>	calculated
Deep drainage	2.8	calculated
Extraction from subsoil	nil	

The only magnitude in this example which is not known and has to be deducted is $n = 1.15$. This is done by drawing a graph as in fig. 8, with axes representing $\log v_r$ and $\log n$. In this graph a curve has to be constructed

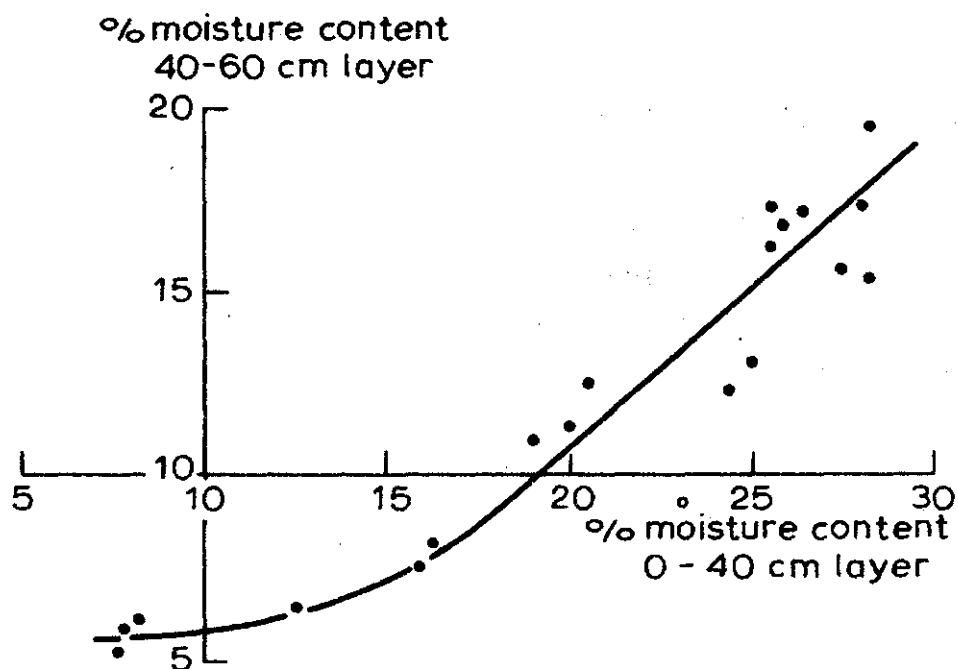


Fig. 7. The moisture content in successive layers bear a certain relation to each other, which can be used for adjustment of erroneous or lacking data, for assessment of accuracy of determination and for obtaining some indication as to the shape of the desorption curve

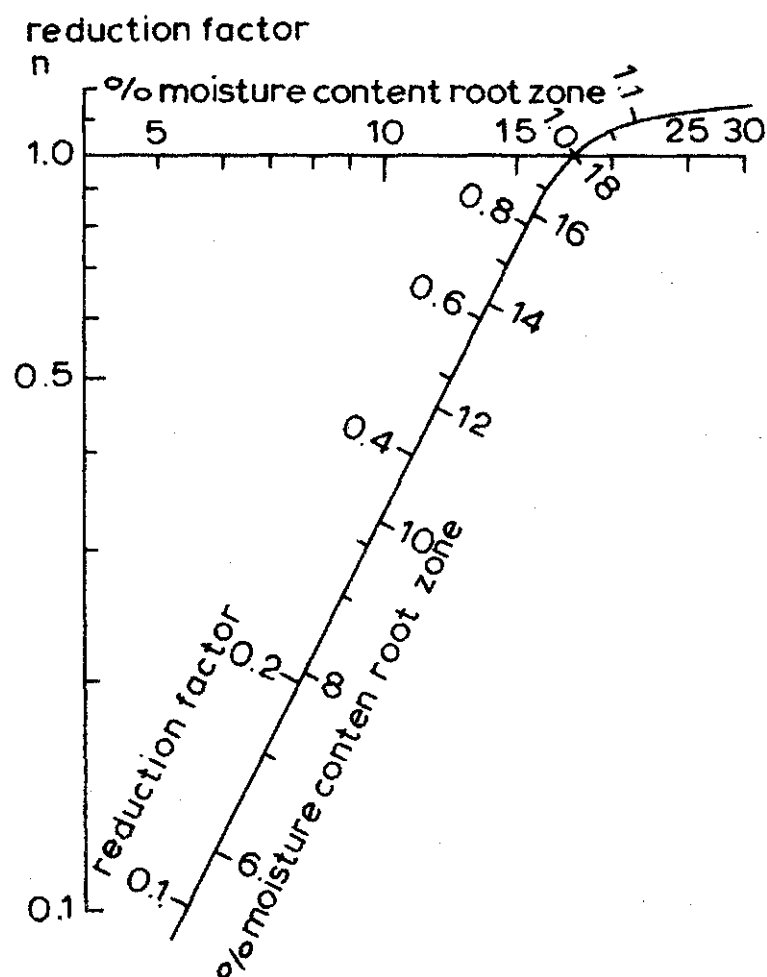


Fig. 8. The constants of the evaporation function are assessed by plotting the $\log E_r/E_o$ values against the log of the soil moisture contents and determining the intercept of the two asymptotes and the inclination of the oblique asymptote

consisting of a sloping and a horizontal asymptote, linked together with a curved transition. It is known that the angle of the sloping part often is of the order of 3. The point of intersection of the two asymptotes is not difficult to assess. Periods with abundant rain will show the maximum value of n and the lower limit of the moisture range over which the climatological situation alone determines the rate of evaporation is found by plotting all the intervals with abundant rain along the moisture content axis and determining the lower limit of these data. This lower limit of the moisture contents along the horizontal line constitutes the upper point of ^(the) sloping asymptote.

Then the slope of the line of reduced evaporation is found by a step by step procedure. First it is assumed that at wilting point some 10 to 20% of the potential evaporation will still be possible as evaporation from the soil. The transpiration by the plant will at this low moisture content have become negligible. With this second point of the sloping line the first approximation of the curve in fig. 8 is drawn, and the data of the series of moisture content observations are processed as in the example of calculation just discussed.

Then the results for the deep drainage and extraction are considered by inspection as to their acceptability. If an indication is found that the data could improve by increasing or decreasing the rate of evaporation, then the curve of fig. 8 is slightly altered till acceptable deep drainage and extraction values are obtained. As the rainfall and storage are known, data for the deep drainage and the evaporation data complete the soil moisture balance.

The water balance, derived from groundwater depth observations

The assessment of the water balance terms starts with splitting the data up in groups with increasing values for the difference in the level of groundwater z_g and water in the ditch. For each group the data for the winter, when the evaporation is small, or the wet season, when the evaporation is a constant fraction of the pan evaporation, are plotted by marking along the horizontal axis the values Δz , the difference between successive groundwater readings, and along the vertical axis rainfall R minus an estimated value for the real evaporation E_r^* . If the amount of water stored is described as $\mu \Delta z$, the storage coefficient μ multiplied with the change in water level Δz , then the water balance equation becomes:

$$(R - E_r^*) = D + \mu \Delta z$$

As shown in fig. 9 the values $(R - E_r^*)$ and Δz are approximately linearly related and the value of D and μ can be assessed for each group ($z-S$). By plotting D against $(z-S)$ and μ against z_g , as depicted in fig. 10, an

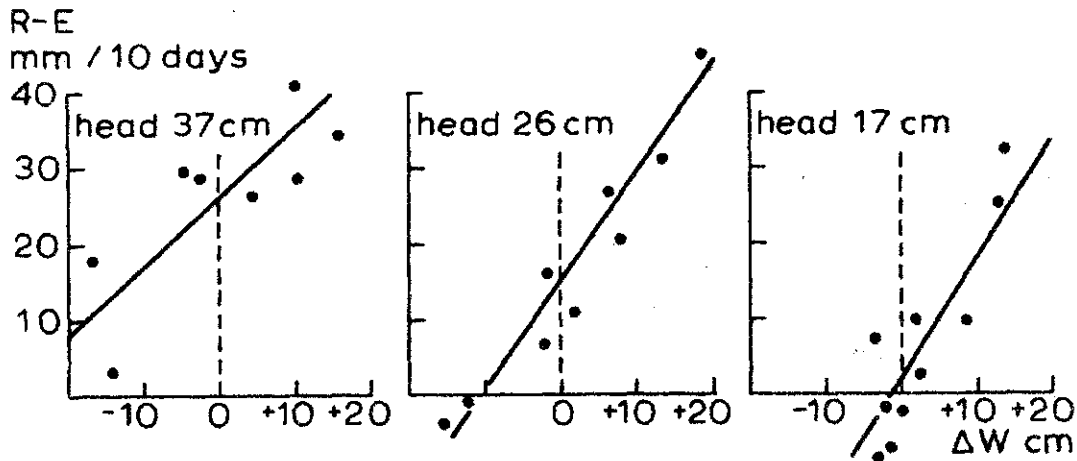


Fig. 9. The preliminary elaboration of groundwater depth observations is carried out by plotting for winter observations rainfall minus estimated evaporation against the change in groundwater depth. The intercept with the $\Delta W = 0$ line represents the discharge, the inclination the storage coefficient

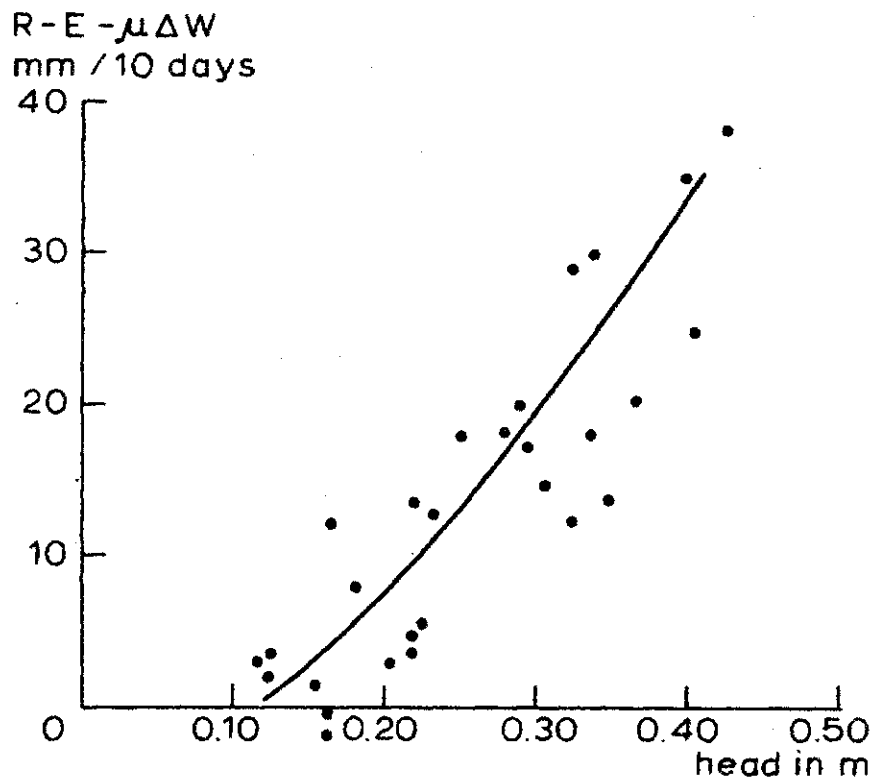


Fig. 10. By plotting winter rainfall minus evaporation and storage against the pressure head, the drainage discharge curve, valid for winter and summer, is obtained

average relation is constructed. This first approximation gives first values for E_r and μ .

The next step is, to calculate the discharge D for each time interval and plot for each month $(R-D)$ against Δz according the equation:

$$(R - D) = E_r + \mu \Delta z$$

In this way for each month an average value of E_r is obtained as represented in fig. 11.

The adjustment of E_r is somewhat complicated. The values of μ plotted against z_g provide an indication of the air content for a depth of the groundwater level equal to z_g . The moisture content can be calculated as pore space P minus the storage coefficient μ . The relation between E_r and the moisture content in the root zone equal to $v_r = P - \mu$ is then calculated as described for the elaboration of the data in the paragraph for the water balance derived from moisture contents.

A few difficulties have to be overcome with the elaboration of groundwater readings. Often the observations are lacking in accuracy. A reading error of 5 cm on a depth of 1.50 m seems trivial, but at this water depth the value of μ may be 0.2 or more and the small error in depth means an error of 10 mm in evaporation and makes the observation useless. Water depth recorders may simplify considerably the adjustment of the data and the solution of E_r and μ .

A second point is the complex character of the relations between storage as well as evaporation on one hand and groundwater depth, capillary movement of the water, evaporative capacity of the atmosphere and influence of the stand of the crop on the other. As an approximate knowledge of the water balance often is sufficient as a basis for the design of a project, these difficulties may be of a minor nature. If, however, a higher accuracy is desired or if moisture conditions have to be reconstructed for moments for which no observations are available, then the results of a first approximation have to be adjusted with the help of the available physical functions and with further supporting observations.

So is the curve, obtained by plotting μ against z_g for a period with no evaporation practically identical with the lower part of the desorption curve and could be supported by laboratory determinations of the desorption curve. The difficulty in separating the storage from the evaporation leads to complicated adjustment problems. A simultaneous determination of the depth of the groundwater level and the soil moisture contents provides data for the storage and may simplify the elaboration of the data considerably.

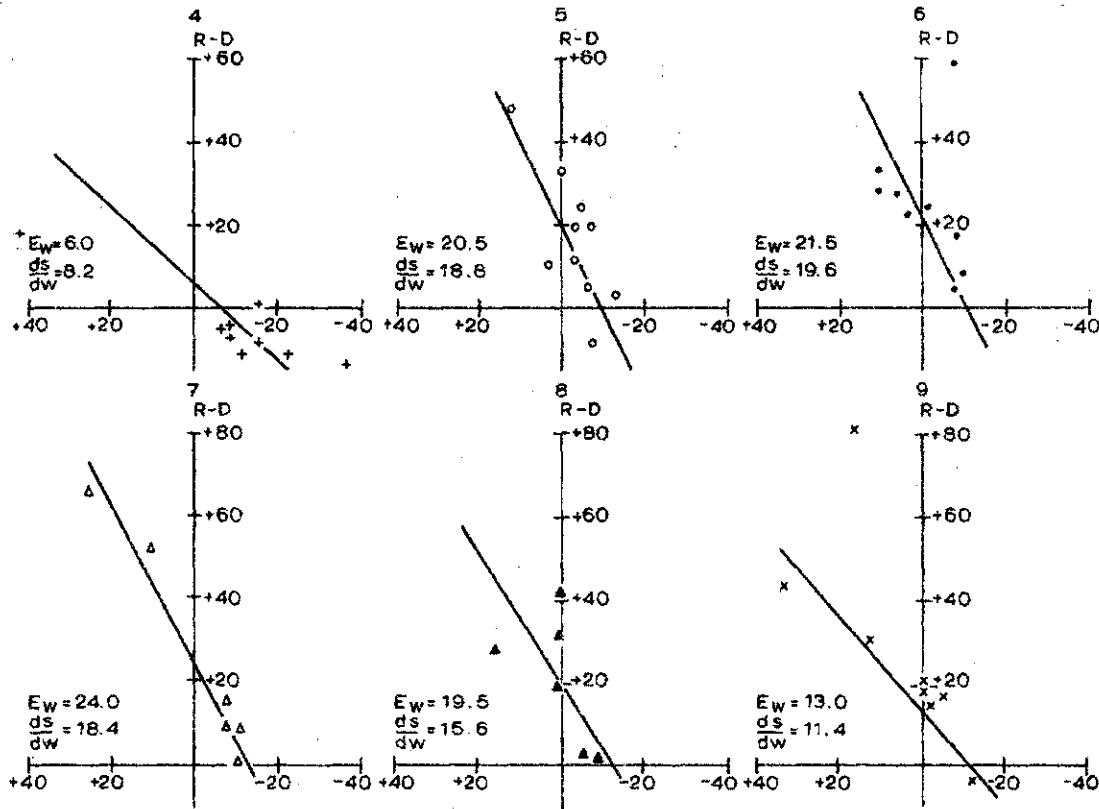


Fig. 11. The rainfall excess $R-D$ is plotted against the change in ground-water depth ΔW for the summer months April (4) to September (9). The evaporation E_w as intercept and the storage capacity $\frac{dS}{dW}$ are found

For research workers which have computer facilities available, the elaboration can be done entirely mathematically. The formula for adjustment in its simplest shape is:

$$R = az_s + bz_s^2 + cz_s^d \Delta z + gE_p + f$$

Here $az_s + bz_s^2$ represent the drain discharge, the cz_s^d represents part of the desorption equation and is equal to the storage capacity μ . The exponent d has to be chosen for the adjustment and changed till the smallest mean error is obtained. The value of g has to be taken constant over the year and is a rather crude approximation. If data on rainfall, pan evaporation and groundwater measurements are available over several years, then a monthly g as an average over the years can be obtained. If a precise result, dependent on the moisture content of the soil is desired, then also data about the density of the canopy and the length of the crop should be available. The adjustment will become difficult and enters the province of fundamental science at which this article was not aimed.

Methods of curve fitting and assessment of parameters

The benefit to be derived from using physical relations is firstly the possibility to obtain acceptable results from data of restricted accuracy and secondly the calculation of the water balance from generally available data as rainfall and calculated evaporation. Often it is supposed that formulae only make sense if the data are accurate, but in such a case a graphical representation is far more advantageous, because it can more easily be grasped, interpolated and used for assessing the value under different conditions.

For inaccurate data, the formulae are used to assess what scale functions should be selected along the axes to ensure the easiest fitting of curves through the scatter diagram. The following example can give an insight into the construction and use of an often applicable nomogram to handle all adjustments to formulae that can be reduced to the type:

$$ax + by + cz = 1$$

Between wilting point and a moisture content that approximates field capacity, a relation exists which defines the real evaporation E_r by a formula of the type:

$$E_r = A (v^{\ell} - v_{wp}^{\ell}) \quad \text{or with } Av_{wp}^{\ell} = C$$

$$\text{or} \quad (E_r + C) = Av^{\ell}$$

The assessment of the exponent ℓ requires the use of logarithms. This transforms the equation to:

$$\log(E_r + C) = \log A + \ell \log v$$

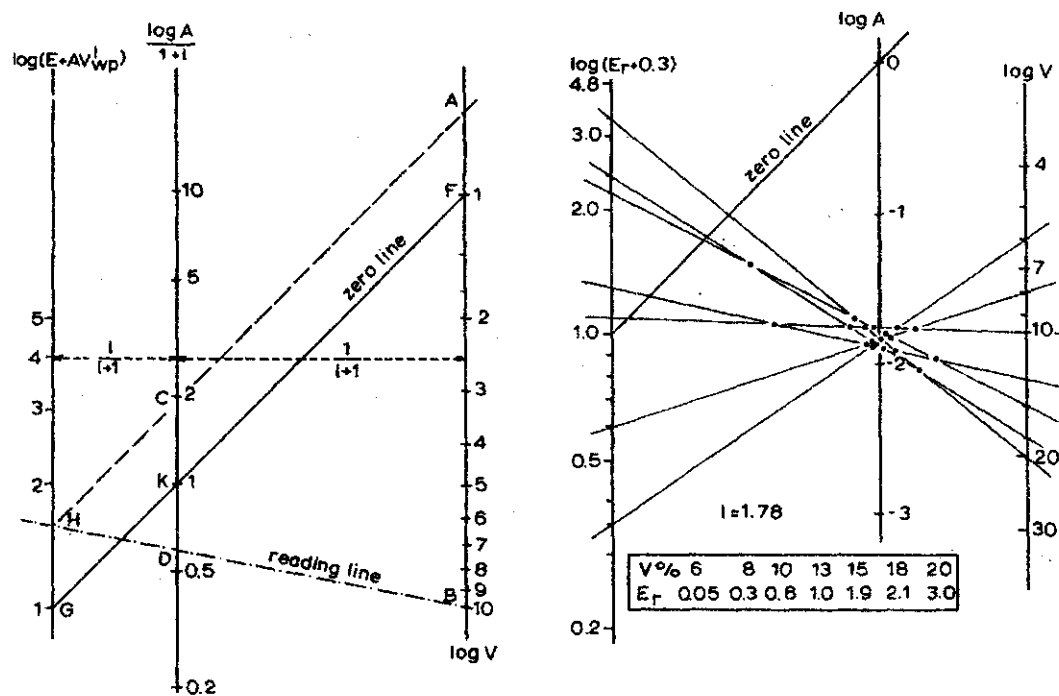


Fig. 12. Nomogram of the formula $E_r = A(v^l - v_{wp}^l)$ for calculation as well as adjustment of the constants. Example of the use of nomograms for the assessment of constants

In this equation the adjustment becomes complicated, because of the position in the formula of the wilting point influence v_{wp}^{ℓ} . Often this effect can be neglected, because it is only of importance in case of low evaporations. In wetter climates the dry conditions are often of restricted practical importance.

The formula can be represented graphically on three parallel axes. On the first axis is plotted $\log(E_r + Av_{wp}^{\ell})$, on the second $\frac{\log A}{1+\ell}$ and on the third $\log v$. The distances between the axes are $\ell/(1+\ell)$ and $1/(1+\ell)$ as shown in fig. 12, on the first and third axis one unit on the scale represents for instance 10 cm and on the second axis $1/(1+\ell)$ units. The zero line linking the $\log(E_r + C)$ scale at the value of $E_r + C = 1$ with the $\log v$ scale at the value $v = 1$ may be given any desired slope by shifting the zero points along the axes and is chosen in such a way that the values on the E-axis and those on the v-axis are marked at about the same height.

If in fig. 12 the point on the E-axis is H, and the one on the v-axis is B, then the following relation holds for GH equal to AF equal to CK:

$$\frac{AF + FB}{1} = \frac{CK + KD}{\ell/(1+\ell)} \quad \text{or} \quad \frac{\log(E_r + C) + \log v}{(1+\ell)} = \frac{\log(E_r + C) + \log A/(1+\ell)}{\ell}$$

$$\begin{aligned} \ell \log(E_r + C) - \ell \log v &= \log(E_r + C) + \ell \log(E_r + C) + \log A \\ \log(E_r + C) &= \log A + \ell \log v \end{aligned}$$

If $\log(E_r + C)$ is plotted with a few well chosen values of $C = Av_{wp}^{\ell}$, and $\log v$ is plotted, the fact that A is a constant ensures that all the reading lines - linking the E_r and v-values which belong together - will intersect in one point situated on the $\log A$ -axis at a point equal to $\log A$. The distance along the $\log A$ -axis to the zero-line is now measured. If a certain value of C was chosen which is not the correct one, or in case the observations are not absolutely accurate, the points of intersection will scatter, but the mean point of this scatter will be an estimate for the position of the $\log A$ -scale and will provide an approximation for ℓ , A and v_{wp} .

An example is given in fig. 12 for the following series of observations:

v %	6	8	10	13	15	18	20
E_r mm	0.05	0.3	0.8	1.0	1.9	2.1	3.0

The observations are plotted with the same logarithmic scale along the two axes, $(E_r + C)$ is plotted in an upward, v in a downward direction. The value of $C = Av_{wp}^{\ell}$ is chosen arbitrarily. The zero line is chosen with such a slope, that the reading lines form a more or less symmetric sand glass

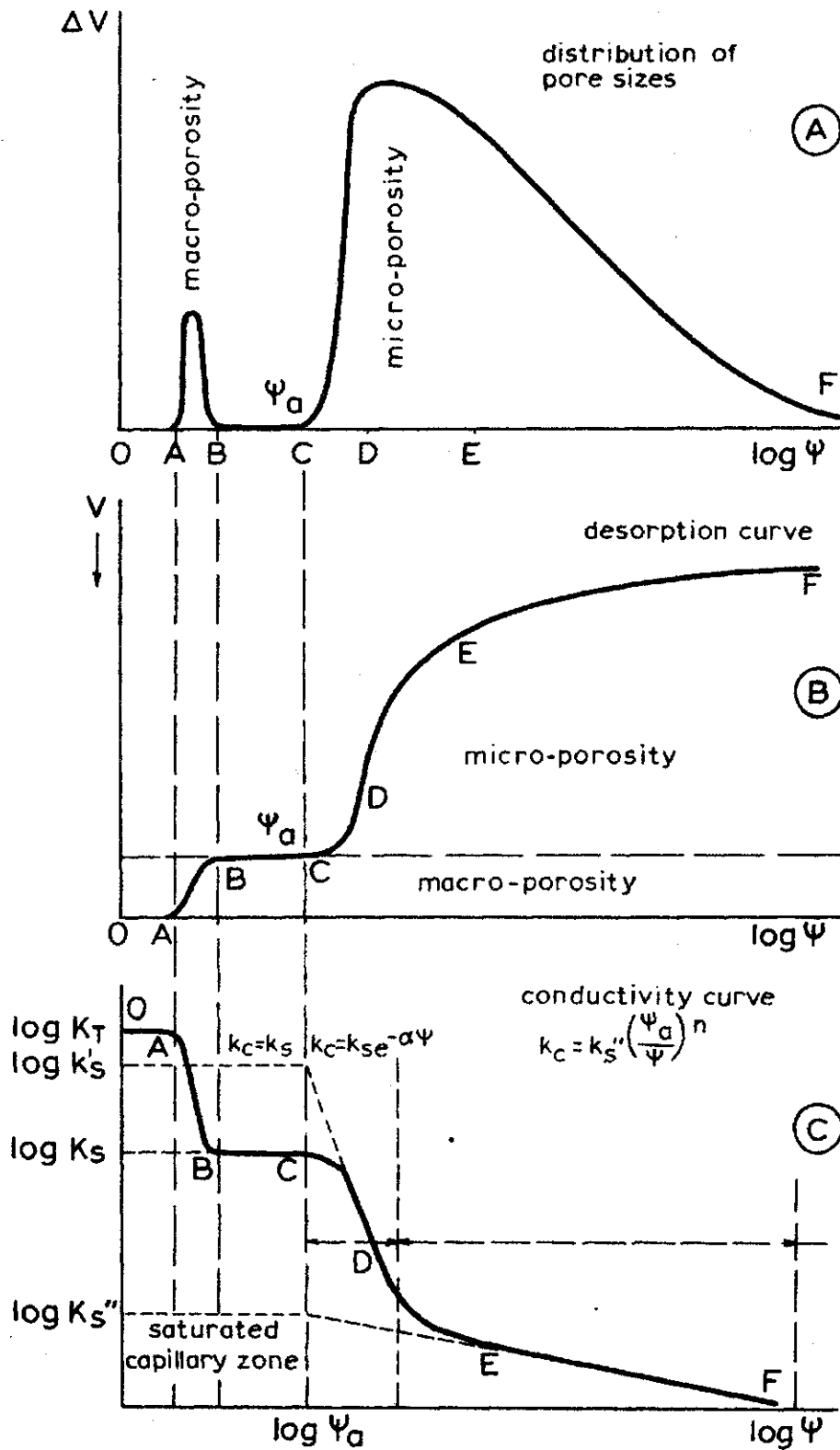


Fig. 13. Explanation of the qualitative relations between the curves for the pore size distribution, the desorption curve as integration of the latter and the conductivity curve as influenced by macro- and micro-porosity and the interjacent pore size range with zero pore probability

shaped image. The points of intersection are marked, 21 in number for 7 pairs of observations and in horizontal and vertical direction the position of the 11th point in horizontal and vertical order is marked as coordinates of the average. Through these coordinates the third axis is drawn and the point of intersection with the zero line is determined. Now the distances DK and $l/(l+1)$ can be measured and the values of l , A and v_{wp} can be calculated.

Is this construction made for different values of C, then the l and A can be assessed which produce the most credible value for v_{wp} . The following table shows what kind of result can be expected:

C	=	0.0	0.1	0.2	0.3	0.5	1.0	3.0	5.0	6.5
l	=	2.45	2.41	1.99	1.78	1.32	1.10	0.515	0.346	0.282
A	=	$182 \cdot 10^{-5}$	$245 \cdot 10^{-5}$	$871 \cdot 10^{-5}$	$155 \cdot 10^{-4}$	$537 \cdot 10^{-4}$	0.134	1.14	2.58	3.78
v_{wp}	=	0.0	4.67	4.84	5.30	5.38	6.17	6.53	6.79	6.82

The result shows that a small range of v_{wp} -values, together with sets of interdependent values of l and A, covering a wide range, are able to give a good adjustment of the data. That the value of C is difficult to assess means, that the point of zero evaporation occupies a very insensitive place in the plant - soil moisture relation.

The relations in the unsaturated zone

The relations in the unsaturated zone are described by two characteristics, the desorption curve, defining the moisture content - stress relation, and the conductivity curve, which gives the relation between the capillary conductivity and the moisture content. Behind these curves are the curves for the pore size distribution, as given in fig. 13. The pore size distribution is a two apex curve, because the pore sizes are related to two origins. The large pores are the result of cracks or root holes, the small pores are the voids between the soil particles. The two origins of pore size distribution show up in two frequency curves, which in fig. 13A are drawn as separate probability distributions, but they may overlap.

The desorption curve is the integration curve of the pore size distribution. The shape of this curve is given in fig. 13B. The curve in fig. 13A consists of three parts AB, BC and CE, and in fig. 13B these three ranges can also be distinguished. In the distribution curve a certain pore size will define the largest pore present. This maximum size will be of less than infinite radius.

Because between the capillary stress ψ and the pore radius a relation exists reading:

$$\psi = \frac{a}{r}$$

the presence of a maximum pore size means that a minimum value of ψ can be distinguished, which can be used as indication of the presence of this maximum pore size. This is the air entry stress ψ_a , the stress at which the first quantities of air start to enter the soil, provided that no macro-porosity is present. If macro-porosity has to be taken into account, there are two of these points, the air entry point A for the macro-pores and point C for the micro-pores. The point A is, however, situated at a moisture stress that is too small to be determined. The air entry point determined in the laboratory conforms with point C. The volume of the macro-pores generally is also small and will normally be overlooked. It is, however, because of its larger pore diameter of decisive importance for the permeability.

The law of Poiseuille states that the amount of water flowing through a capillary pore is proportional to the fourth power of the pore diameter. The macro-pores therefore contribute largely to the conductivity, the micro-porosity contributes far less.

The capillary conductivity is the sum of the contributions of all the pores which contain water and allow moisture to flow. These will only be pores below the size which just can hold its moisture against the stress resulting from the degree of saturation. The curve for the capillary conductivity will follow from the pore size distribution. The size has to be brought to the fourth power and these values have to be integrated to give the conductivity curve, as depicted in fig. 13C. In the range OA and BC, where no pores are present, the conductivity will not increase. In the range AB, where in fig. 13B the increase is small, the conductivity will increase quickly, in the range EF this is just the reverse as a consequence of the small pore sizes.

Taking into account that the range OB is too small to allow the determination of ψ -values - capillary stresses of a few mm cannot be determined in undisturbed samples of a few cm thick - one will find for the conductivity curve a simplified shape. The saturated conductivity in fig. 13C will coincide with point K_T . The highest unsaturated conductivity will be situated on the line BC where, dependent on the soil, a certain constant value will be found. In the range CD, a more or less straight line relation will be found for the lower end if $\log K_c$ is plotted against ψ . In the range EF also a straight line can be constructed, but now if $\log K_c$ is plotted against $\log \psi$. The precise nature of the relation between the desorption curve and the conductivity curve is not known. The Poiseuille equation was mentioned, but does not correctly allow for the effect of the variable diameter of the pores, for the

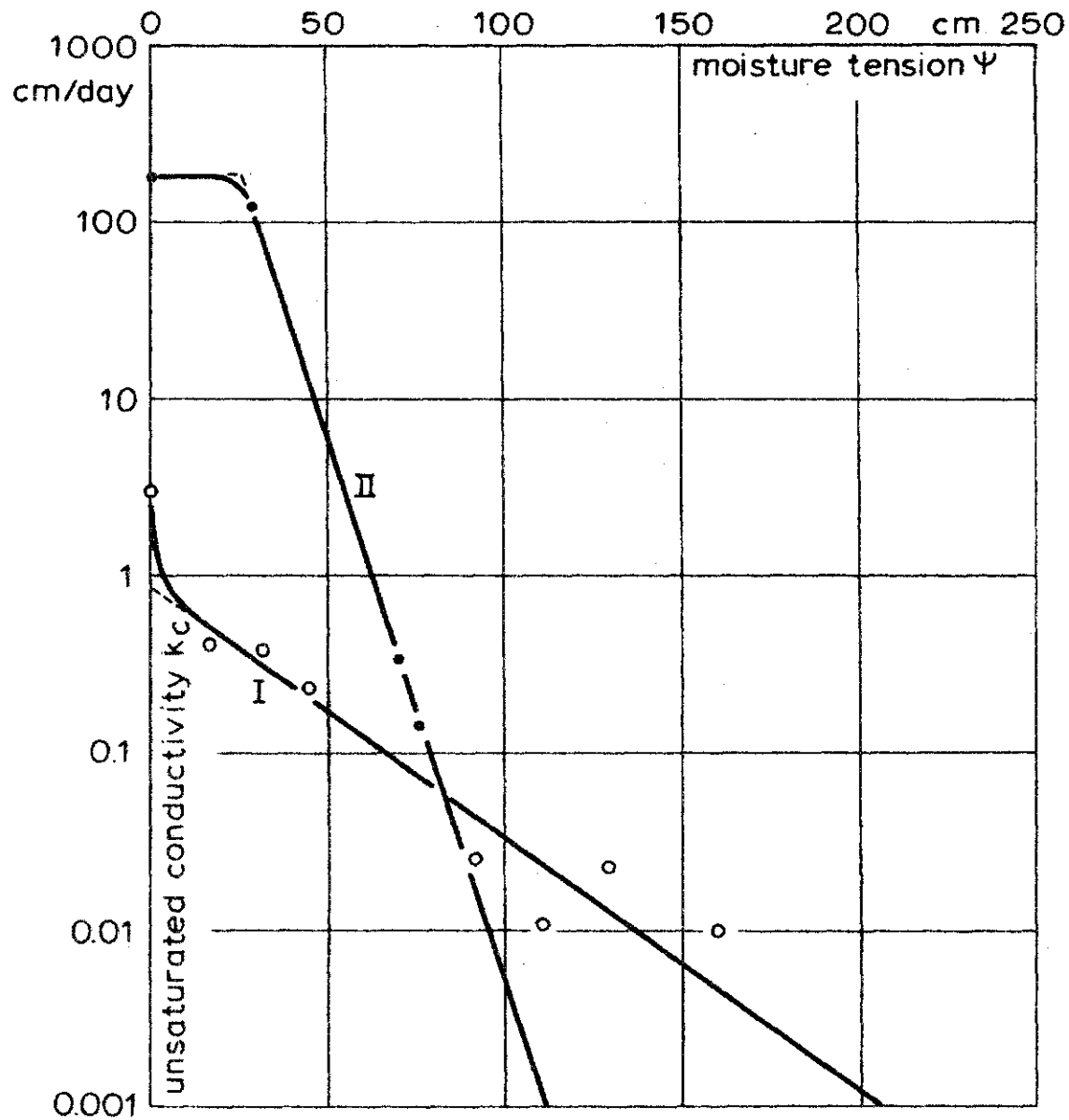


Fig. 14. Example of the possible aspects of the curves for capillary conductivity. Curve I has a low zero stress capillary conductivity and influence of macro-porosity. Curve II has a high zero stress capillary conductivity but a clearly indicated saturated capillary zone

tortuosity and the ramification of the pore system and for the presence of air blocks. The Poreseuille relation, however, is important as a basis of explanation why the small volume of macro-pores often accounts for the larger part of the saturated conductivity, and why the saturated conductivity, plotted against the moisture stress, often has no clear position on the curve for the unsaturated conductivity. In fig. 14 an example of a laboratory determination of the curve for the unsaturated conductivity is given.

The desorption curve

The data for moisture content and moisture stress, if analysed in detail, provide not only information for the pore size distribution. If the air entry point is assessed and the total pore space is measured, and further a fluent line is drawn or calculated through the points depicting the stress and moisture content observations, then four desorption characteristics are available, which are of interest. Fig. 15 explains the pore space characteristic.

One can distinguish the capillary pore space P_a which goes with the air entry stress ψ_a . The non-capillary pore space ΔP_s is the difference between the capillary and the total pore space P_T . The mathematical pore space P_M is the value of P which gives the best fitting fluent line, dependent on the presence of the macro-porosity as distinguished in fig. 13, and is higher than the value for P_a but can be smaller as well as larger than P_T . The value of P_T is the value of the pore space determined in the laboratory. The difference $P_T - P_M = \Delta P_s$ is positive in case macro-porosity is present. In case the micro-porosity P_s is zero, the laboratory analysis provides a positive difference $P_M - P_a = \Delta P_a$. In case this positive value is obtained, the air entry stress ψ_a can be deduced from the desorption formula and compared with the laboratory determination. Often the laboratory determination proves to be somewhat higher.

The desorption formula, of an empirical nature which holds well for undisturbed not swelling soils, is:

$$\psi = \frac{G (P_T - v - \Delta P_s)^n}{v^m} \quad (5)$$

$$\text{or} \quad \log G - \log \psi = m \log v - n \log (P_T - v - \Delta P_s)$$

$$\text{The air entry value is } \psi_a = \frac{G \Delta P_s^n}{v^m}$$

In the following discussion, the distinction between ΔP_s and ΔP_a , only differing in sign, will be neglected.

The formula will often be used in the shape of

$$b (G^* - pF) = p \log v - (1-p) \log (P^* - v)$$

$$\begin{aligned} G^* &= \log G \\ P^* &= P_T - \Delta P_a \end{aligned}$$

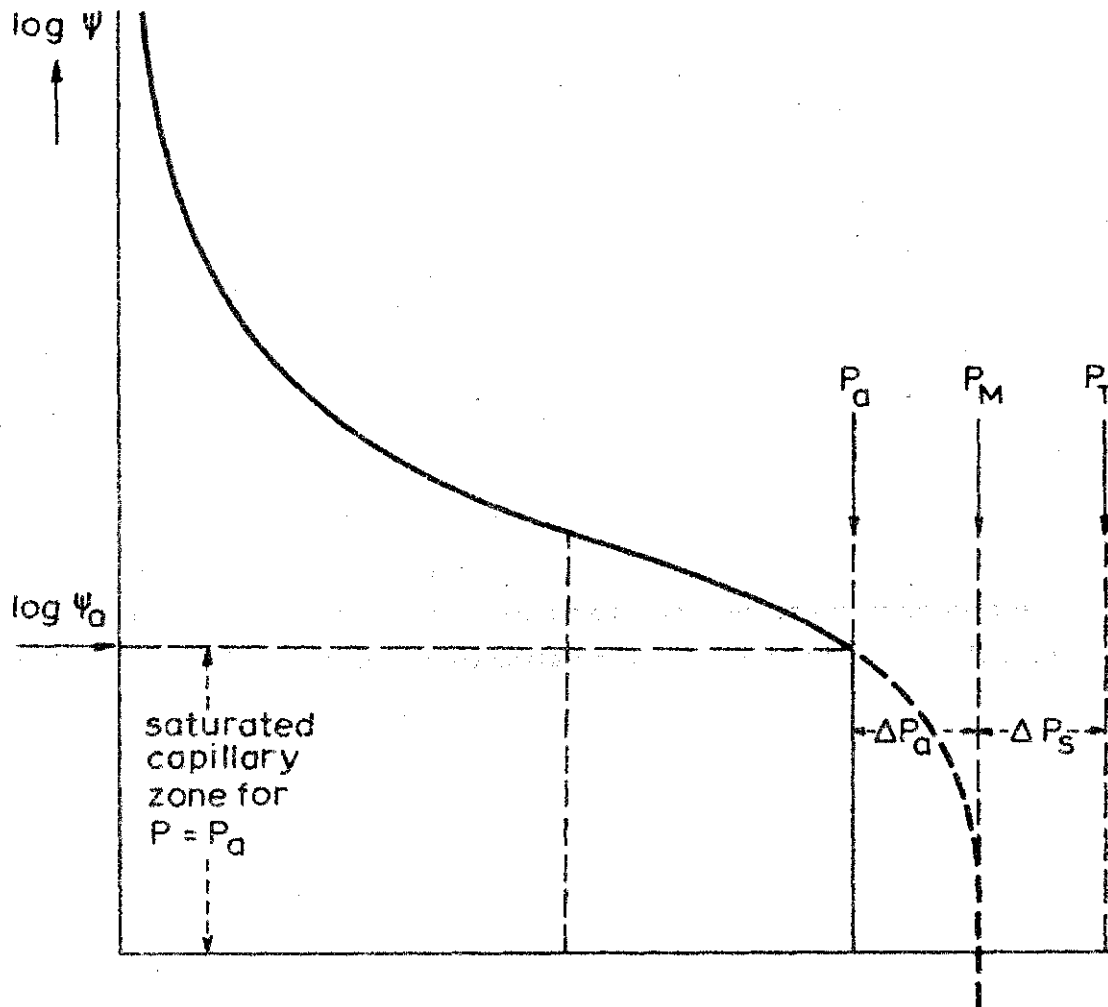


Fig. 15. Influence of the presence of a saturated capillary zone with pore space P_a and of macro-porosity with pore space P_T on the desorption curve and the position of the pore space P_M found by mathematical adjustment. ΔP_s can be positive or negative but its negative value cannot exceed ΔP_a .

As will be shown, a major problem with respect to the use of the formula is to obtain in an easy way an indication of the value of $(P_T - \Delta P_S)$ and of ψ_a , a difficulty which arises from the inaccuracies in the determination of the data.

Assessment of the constants

For the solution of the desorption equation as well as for curve fitting three solutions are available.

The first and second solution use the nomogram with three parallel lines, see fig. 16A and B. In fig. 16A the right x-axis represents in an upward direction the $\log v$ -values, but is marked with the values of v itself. The left y-axis is calibrated in a downward direction with $\log(P^* - v)$, but also indicated with v -values. In this diagram the reading lines A-A' and B-B' for $\log \psi$ -values of 1.5 and 6 are drawn. These lines link the points for $v = 2.4$ and $v = 47.5\%$ moisture, which are the moisture contents obtained by analysis going with the stresses $\log \psi = 1.5$ and 6.0 . If other, but identical moisture content values on the x- and y-axes are connected, then at the intercept with the z-axis the matching $\log \psi$ -value is indicated.

When constructing the nomogram, the position and calibration of the z-axis is, however, not yet known. The construction is as follows:

The vertical distance between the two reading lines has to be divided in the $\log \psi$ intervals of the observations. The line for $\log \psi = 3$ is drawn at $(3 - 1.5) / (6.0 - 1.5)$ - or $1/3$ of the distance between the two lines - from the A-A' line, the line for 4.2 is situated at $(4.2 - 1.5) / (6.0 - 1.5)$ part of the distance between the lines from the A-A' line. This system of lines is indicated in the nomograms of fig. 17 as the ψ -lines.

Next a second system of lines is constructed, indicated in fig. 17 as v-lines, which link the v -values on the two axes which match with the successive $\log \psi$ -values. The points of intersection of the drawn ψ -line and the dashed v-line for the values of the analyses are marked with a circle and through these circles the vertical $\log \psi$ -axis is visually adjusted. The intersection with the A-A' line is marked with $\log \psi = 1.5$, the intersection with the B-B' line with $\log \psi = 6.0$. The ψ -scale is constructed as a metric scale between these two points, the value of b is calculated as the ratio

$\mu_\psi = L / (6.0 - 1.5) \mu_v$ in which L is the length along the $\log \psi$ -axis between the A-A' and B-B' line, μ_v is the scale unit of the $\log v$ and $\log(P^* - v)$ axes and $(6.0 - 1.5)$ is the number of $\log \psi$ -units between the A-A' and B-B' lines. The nomogram can be used for fitting the formula to the observations, but with the three axes correctly calibrated, each reading line linking

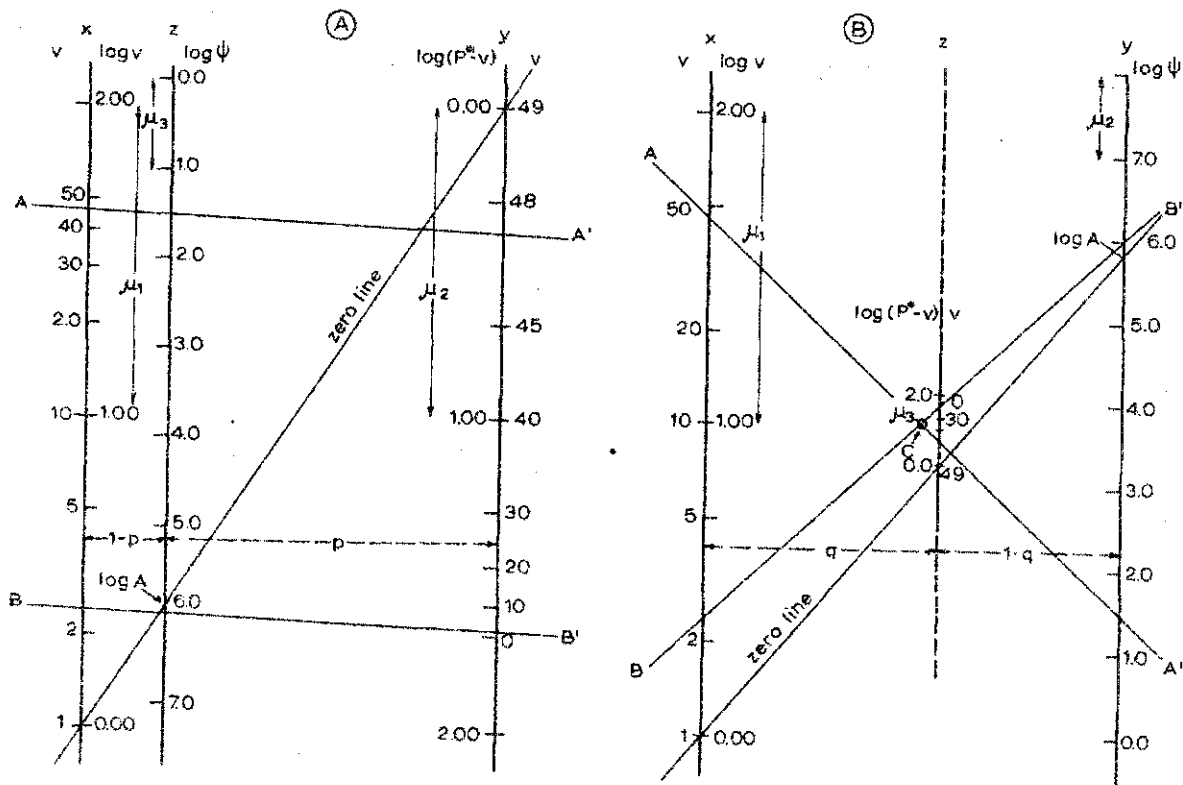


Fig. 16. Construction principles of the nomograms for the formula $b(G - \log \Psi) = p \log v - (1-p) \log (P-v)$ for calculating Ψ or v and adjustment of the constants b , G , p and P . The nomogram in fig. 16B is more specially advocated for assessment of P

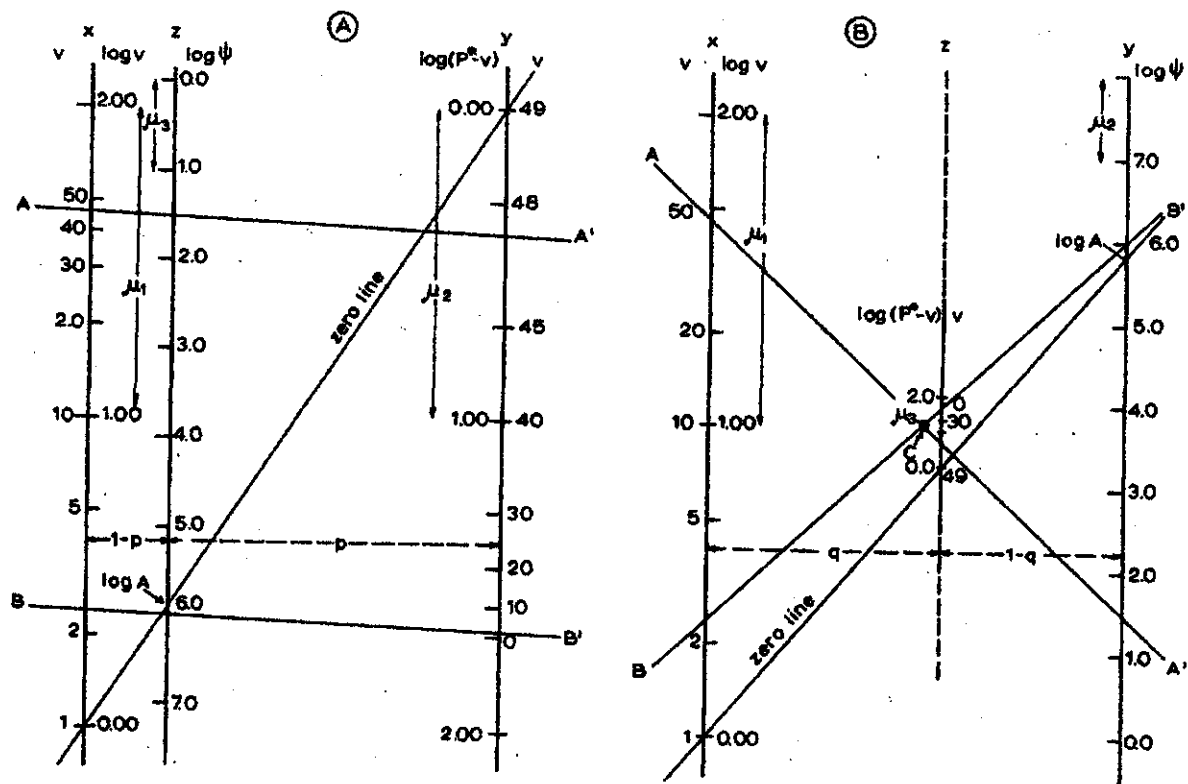


Fig. 16. Construction principles of the nomograms for the formula $b(G - \log \Psi) = p \log v - (1-p) \log (P-v)$ for calculating Ψ or v and adjustment of the constants b , G , p and P . The nomogram in fig. 16B is more specially advocated for assessment of P

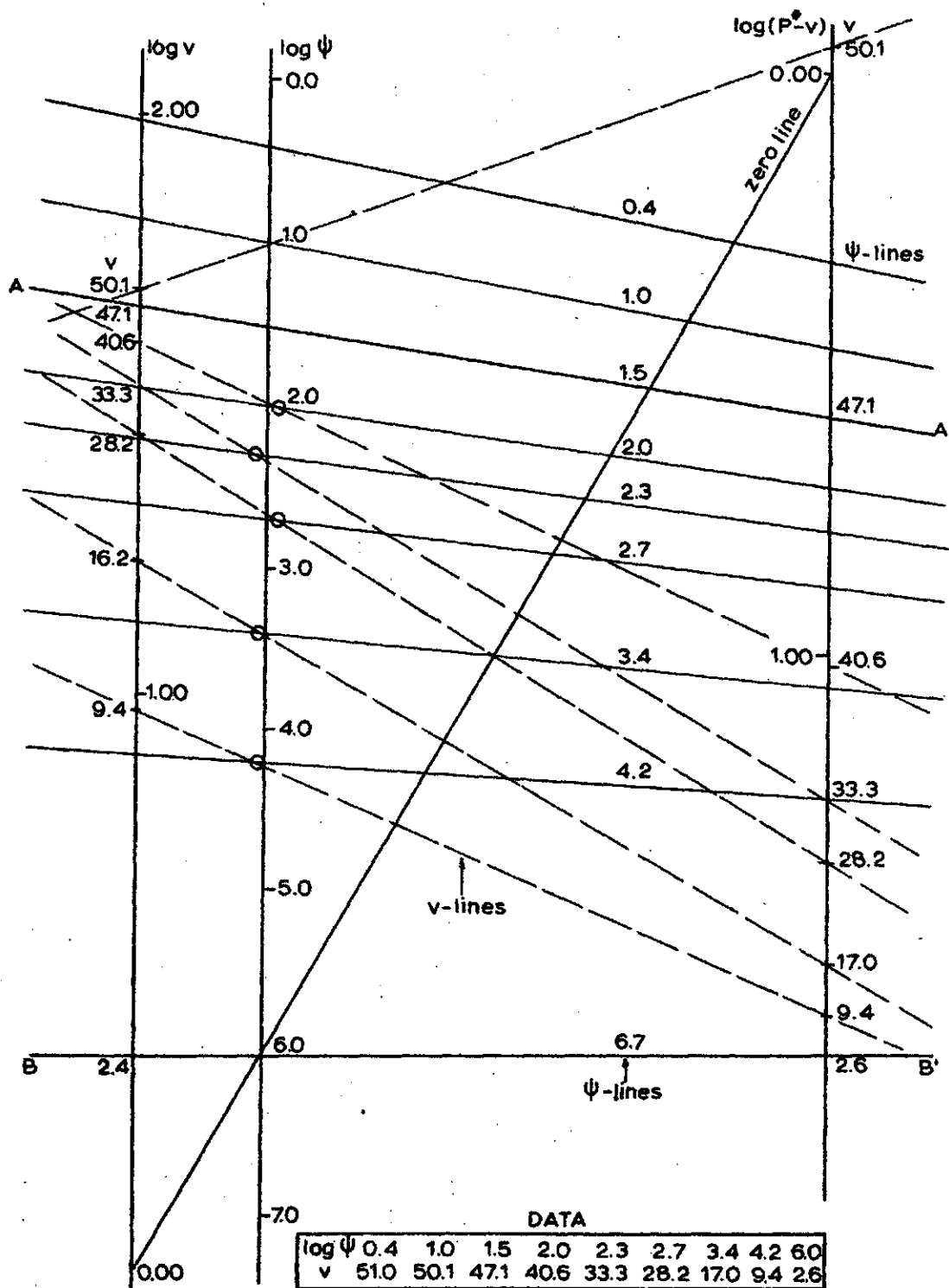


Fig. 17. Example of adjustment of the given data, according to the nomogram of fig. 16A in which for every value of P to be tried a new sheet has to be worked out

two equal values of v on the outer axes with the value of ψ on the middle axis forms a solution of formula 5. In this way the same nomogram is used for assessing the constants as well as for solving the formula.

This nomogram is simple, but has the disadvantage that the assessment of the value of P^* requires the repeated construction of the whole nomogram. The necessity of repeated construction can be evaded by calibrating the outer axes both in upward direction with $\mu_1 \log v$ and $\mu_2 \log \psi$ in which the ratio μ_1/μ_2 is selected in such a way that the observations for $\log v$ and $\log \psi$ cover about the same length of each axis. Fig. 18 shows how this nomogram is made. Often a ratio $\mu_1/\mu_2 = 3$ is suitable. The $\log v$ -axis is again indicated with v . The two reading lines $A-A'$ and $B-B'$ for $\log \psi$ data of appropriate value - here 0.4 and 6.0 - are drawn, which intersect at C. Now between these two lines on vertical distances equal to $\log (P^*-v)$ and plotted at a convenient scale, the values of $\log (P^*-v)$ are marked in such a way that the values of $\log (P^*-v)$ for $\log \psi = 0.4$ and 6.0 are situated on the $A-A'$ and $B-B'$ lines. A set of lines is constructed for successive values of P^* . These lines are in the top right hand side of the nomogram indicated as 10 $\log (P^*-v)$ lines.

Now two sets of lines are constructed. The dashed (P^*-v) lines connect the (P^*-v) values of the analysis as marked on the arbitrary $\log (P^*-v)$ axis with point C. The third set of v, ψ -lines connects the values of matching v and ψ -values and intersects with the (P^*-v) lines. The points of intersection of the matching (P^*-v) and v, ψ -lines - belonging to the same observation - are marked and these markings should be situated on a vertical line. In case the value of P^* is not correctly estimated, then for lower values of (P^*-v) - or higher values of v - the points fan out to the left if P^* was taken too low, or to the right if P^* was taken too high. A new value of P^* is selected and a new arbitrary $\log (P^*-v)$ axis of the set of 10 $\log (P^*-v)$ axes is drawn. Because the distance between the values (P^*-v) to be situated on the $A-A'$ and $B-B'$ lines varies, the new axis will not coincide with the former one and the construction can be repeated on the same sheet of paper, with only the (P^*-v) lines changing. Fig. 18 gives an example of the construction of the successive approximations of the $\log (P^*-v)$ axis.

The intersections of the vertical line through the constructed points with the $A-A'$ and $B-B'$ lines again provide two points of the 3.87 $\log (P^*-v)$ scale and the calibration can be constructed, the magnitude of a scale unit can be assessed, the zero line through the points v and (P^*-v) equal 1.0 can be drawn and at the intersection of the zero line with the $\log \psi$ -axis

the value of G can be read.

The formula can now be calculated by determining:

$$m = \frac{\mu_1(1-q)}{\mu_2 q} \quad \text{and} \quad n = \frac{\mu_3}{\mu_2 q}$$

The values of μ_1 , μ_2 and μ_3 are the scale units for the $10 \log v$ -axis, the $4 \log \psi$ -axis and the $3.87 \log (P^* - v)$ axis. It is advisable to indicate the scale not only with the variable - for instance $(P^* - v)$ - but also with the function - $\log (P^* - v)$ - as well as with the length of the scale unit used - therefore $3.87 \log (P^* - v)$.

The advantage of this nomogram is that on one sheet of paper the solution for a number of P -values can be worked out and comparison of the shape of the F -lines, in fig. 18 obtained. in the successive solutions, allows more readily to make an estimate of the final solution for P .

The disadvantage of the two nomograms is, that in case of rather inaccurate observations, the solution becomes difficult. Inaccuracy of the values for $\log \psi = 6.0$ is troublesome, because the solution is sensitive to this value. The same holds for the lower observation, providing line $A-A'$, for which the value of ψ has to be chosen above the tension of the air entry point and this value is often not known.

A third solution which in the case of inaccurate data may be helpful is the one in which $\log \psi$ is plotted against $-\log (P^* - v)$ as well as against $+\log v$ as depicted in fig. 19. It can be proved, that the line EF , linking the points of intersection of each curve, has relative to the $\log \psi$ -axis an inclination equal to b . The line EF divides each horizontal line GK according to a ratio p to $(1-p)$. The distance AB between the arbitrarily chosen zero points of the $\log v$ and $\log (P^* - v)$ axis, if divided in a ratio

$$\frac{BM}{AM} = \frac{p}{1-p}$$

provides a point of intersection L of a vertical line through M and the oblique line EF . This point L indicates on the $\log \psi$ -axis the value of A . Further, the tangent lines on the curved $\log v$ -line in E and the $\log (P^* - v)$ line in F provide values for the approximation of the desorption curve:

$$\psi = G_1 (P^* - v)^{m_1} \quad \text{and} \quad \psi = G_2 v^{m_2}$$

for the dry and wet ends of the moisture stress - moisture content relation. The last equations are used if a simplified mathematical relation for the

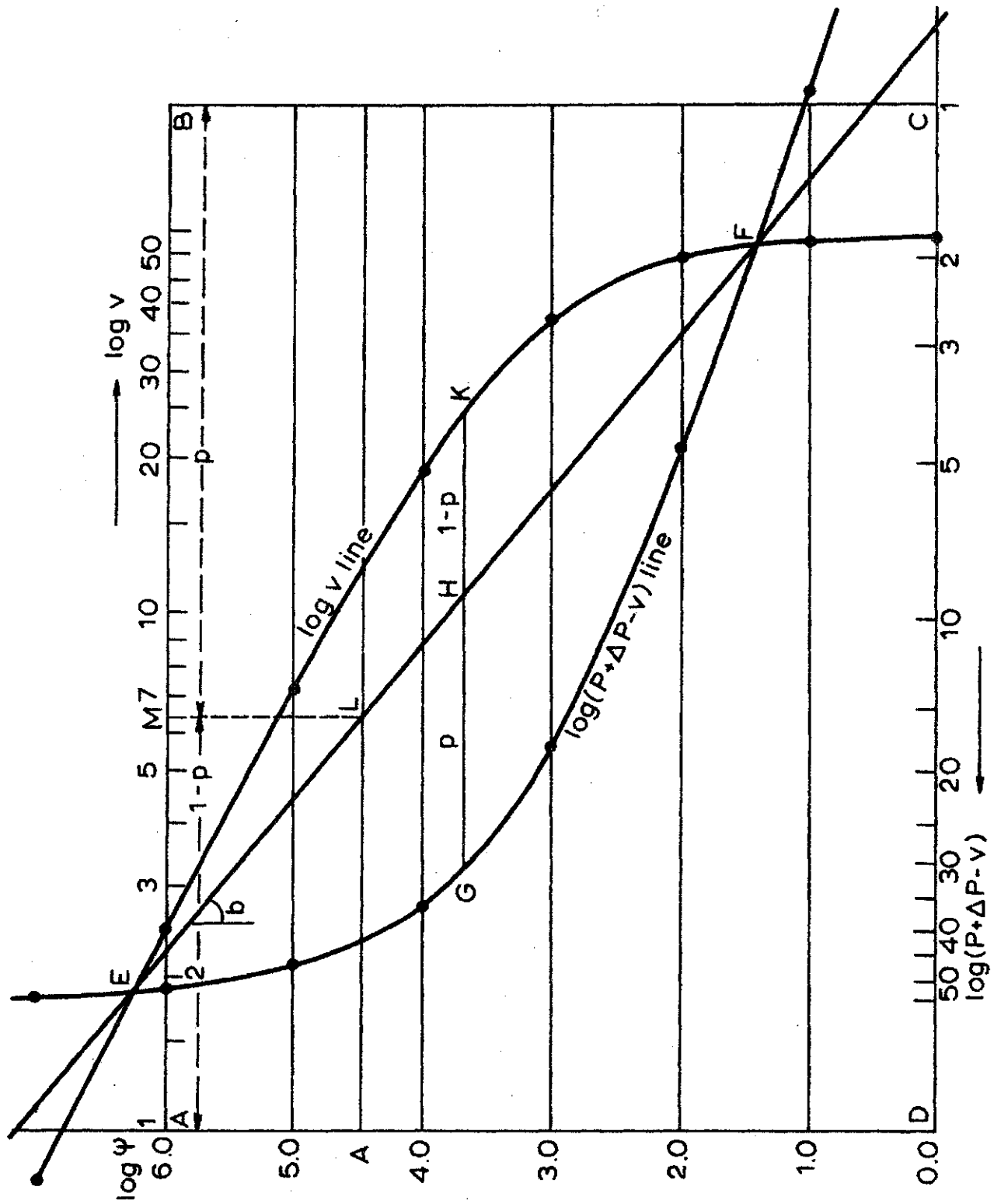


Fig. 19. Example of a method for adjustment of desorption data, which may be helpful in case of inaccurate observations

lower or upper part of the desorption curve is considered acceptable.

The soil moisture profile

The moisture profile of the soil is related to the soil moisture stress that prevails at the point of sampling according the relation that was discussed in the paragraph dealing with the desorption curve. If at any layer of the profile this moisture stress ψ is equal to the height z above the groundwater table, then the moisture distribution in the profile is in equilibrium and no capillary flow will occur.

If any flow is present, this means that the difference $(\psi - z)$ will depart from zero and will vary in upward or downward direction. Moisture will be extracted or stored in the successive layers of the soil and the moisture stress ψ will the more diverge from the values z for the height above groundwater, the higher the rate of flow v_c or the lower the unsaturated conductivity k_c of the soil is. This divergence can be appreciable in case of upward capillary flow and soil moisture extraction. For downward flow the divergence from the equilibrium profile is much smaller and, due to the inconstancy of the rain as compared with the far less variable evaporation, lasts much shorter. The discussion will therefore centre on the upward capillary flow.

The extraction v_c of soil moisture from the profile should be described by a non-steady flow equation. Because, however, these equations are rather cumbersome, the use of an approximation is proposed, assuming that the extraction from each layer is the same. This is described by:

$$v_z = k_c \left(\frac{d\psi}{dz} - 1 \right)$$

$$\frac{dv_z}{dz} = a v_c$$

or
$$v_z = (az + b)v_c = \frac{z + z_0}{z_s + z_0} v_c$$

z_s = the height of the soil surface above the groundwater table
 z_0 = a parameter governing together with z_s the linear increase of the quantity of water that flows upward
 v_z = the flow at the height z
 v_c = flow at the soil surface z_s , often equal to the real evaporation E_r
 k_c = capillary conductivity at height z

This moisture flow is also dependent on the capillary conductivity k_c which may be described by:

$$k_c = k_o e^{-\alpha\psi}$$

The value of k_o is obtained by calculating the value of k_c for $\psi = 0$.

The equation from which the soil moisture profile can be deducted can be written as:

$$\frac{z + z_o}{z_s + z_o} v_c = k_o e^{-\alpha\psi} \left(\frac{d\psi}{dz} - 1 \right)$$

Here $\frac{d\psi}{dz}$ is the gradient in the unsaturated zone, by -1 the gradient due to gravity is given. The differential equation can be solved and after some re-writing becomes:

$$(1 - e^{-\alpha z}) \left\{ \left(\frac{z_o - 1/\alpha}{z_s + z_o} \right) \frac{v_c}{k_o} + 1 \right\} + \left\{ \frac{1}{z_s + z_o} \frac{v_c}{k_o} z \right\} = (1 - e^{-\alpha\psi})$$

The shape of the curves, which follow from this equation, are given in the following table:

		z_o	0	150	∞
Constants		z cm			
$v_c = 0.4$ cm/day $k_o = 20$ cm/day $z_s = 150$ cm		20	20.0	20.2	20.5
		50	50.4	51.2	52.0
		100	104	107	111
		150	188	202	218
		160	244	311	
		166	∞		
		159		∞	
		158			∞

* The assumption of a linear increase of the extraction is a simplification.

The equation can be transformed into:

$$\frac{de^{-\alpha\psi}}{dz} + e^{-\alpha\psi} = - \frac{\alpha v_c}{k_o} f(z)$$

$$\text{or } \frac{dx}{dz} + Px = -Q \quad P = +\alpha \quad Q = - \frac{\alpha v_c}{k_o} f(z)$$

This is a well-known equation which can be solved if $f(z) e^{-\alpha z} dz$ can be integrated.

A quantity v_c of 4 mm/day is flowing upward under influence of the gradient due to moisture extraction at the soil surface. Dependent on the distribution of the extraction over the profile the increase in moisture stress, as compared with the equilibrium stress $\psi = z$, changes. The more the extraction is concentrated in the upper layers of the profile, the higher the moisture at a certain rate v_c of upward flow can be lifted. The difference between z_{\max} for $z_0 = 0$ and $z_0 = \infty$ is 8 cm for $v_c = 4$ mm/day, but this difference becomes larger the lower the rate of upward flow.

If this equation is used for the assumption that the flow of water through the profile has the same value in any layer, meaning that the capillary rise consists of water drawn from the groundwater, the situation is defined by taking $z_0 = \infty$. This results in

$$(z_0 - 1/\alpha) / (z_0 + z_s) = 1$$

$$\text{and} \quad 1 / (z_0 + z_s) = 0$$

and leads to the simplified equation (Rijtema, 1965*):

$$(1 - e^{-\alpha z}) \left(1 + \frac{v_c}{k_0}\right) = (1 - e^{-\alpha \psi})$$

A further simplification is possible. From the last mentioned formula may be derived the expression:

$$v_c = \frac{k_0 (1 - e^{-\alpha (\psi - z)})}{e^{\alpha z} - 1}$$

In cases where ψ is large compared with z , the value of $e^{-\alpha (\psi - z)}$ can be neglected. By putting $v_c = E_w$, the expression for the real evaporation, assumed to be a steady flow, can for conditions in which the availability of the soil moisture is the limiting factor be derived by the use of the equation:

$$E_w = \frac{k_0}{e^{\alpha z} - 1}$$

* Rijtema, P.E.: An analysis of actual evapotranspiration.
Agric. Res. Rep. 659, p. 1-107

In case of values of αz above 2.5, an easy assessment of α and k_0 can be made by plotting $\ln E_w$ against z according

$$\ln E_w = \ln k_0 - \alpha z$$

This is a straight line relation with intercept $\ln k_0$ and tangent α .

If - what is common - v_c is very small compared with k_0 and v_c/k_0 can be neglected against 1, the equation reduces to

$$z \approx \psi$$

This solution shows, that a large value of k_0 results in a moisture stress profile which does not perceptively differ from the equilibrium profile. The rate of capillary rise then depends on the intensity of the real evaporation, which is, as will be shown in the next paragraph, a function of the moisture stress in the root zone.

This situation, for which the equation needs not to be used and only the desorption curve is of importance, occurs often. The cases, however, where the capillary supply of water is small, are agriculturally of importance and the treatment of observational data to solve the parameters, deserve closer attention. In fig. 20 a graphical representation of the equation is given.

The fitting of the data for groundwater depth, evaporation and moisture stress to this formula will be influenced by the method of assessing the values of E and of the storage capacity μ . The value of μ is equal to the air content of the top layer of the soil. Are these values obtained by an analysis of the groundwater depth, then in the equation for the moisture profile z is equal to z_s . Is the analysis done by studying soil moisture determinations from samples taken at successive depth, then z and z_s differ.

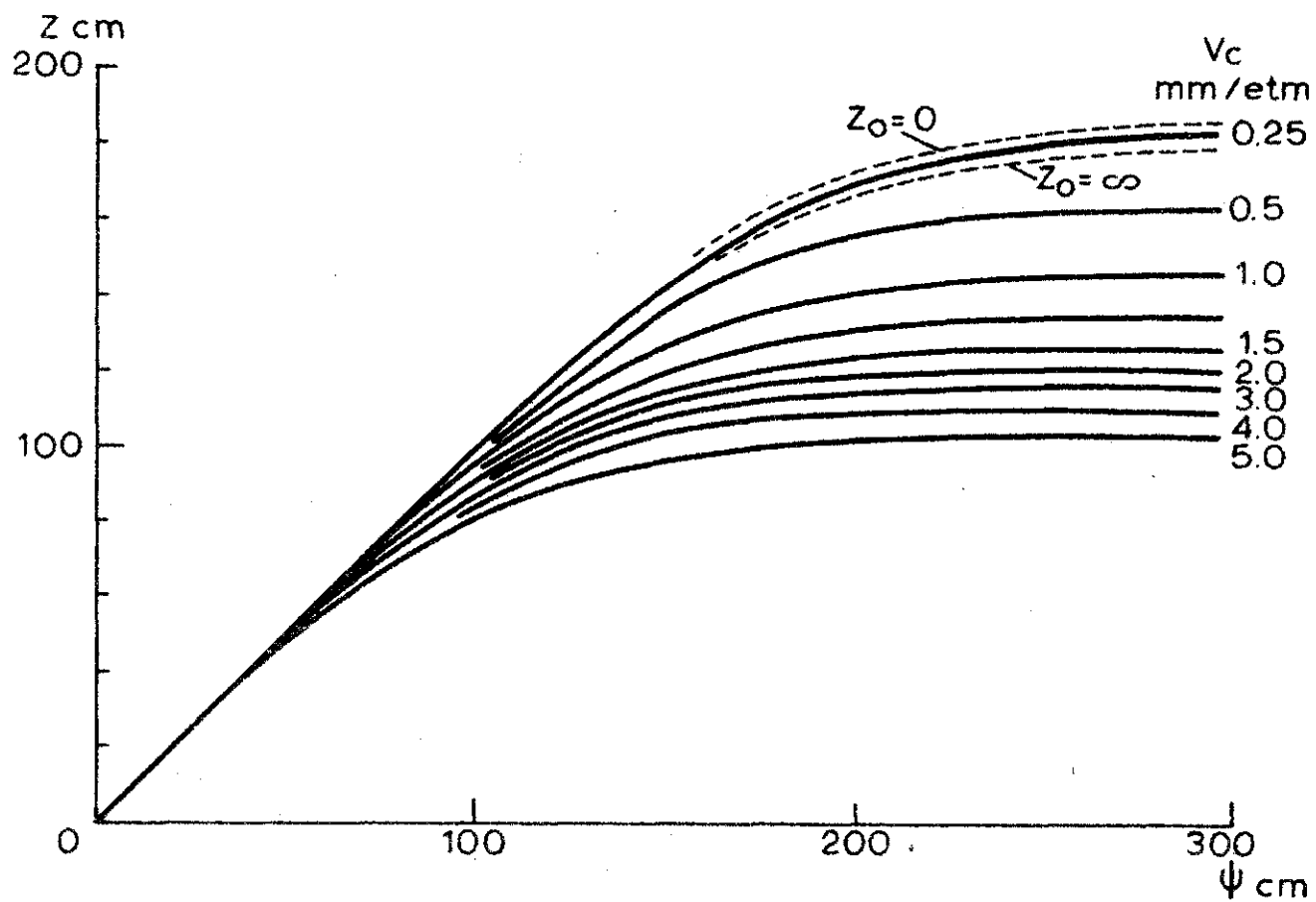


Fig. 20. Example of the relation between soil moisture stress ψ , velocity of capillary rise v_c and groundwater depth z for $k_0 = 10$ cm, $\alpha = 1/30$ and $z_0 = 150$ cm. The dashed lines are for $z_0 = 0$ and infinite

Calculation and assessment of constants

The assessment of the constants can be done with a nomogram, although the nomogram is complicated and the adjustment of the constants is not very efficient. The nomogram has, however, the advantage that no calculation errors have to be feared.

The nomogram consists of 8 parallel axes and 3 oblique zero lines, which also are calibrated. In fig. 21 the nomogram is represented. The significance of the values indicated along the axes is the following:

The scales along the axes are calibrated according to the construction of the formula, but the indication is done with the variables taken up in the formula. For axis 1 for instance, the values indicated numerically are calculated as αz . The calibration, however, is carried out for $20(1 - e^{-\alpha z})$. This axis is located at the zero-coordinate in the horizontal direction.

For each axis, in the table below, the number as indicated in the nomogram is given and the horizontal distance between each axis and axis nr. 1 is mentioned as the horizontal coordinate. In the third column the scale which is marked along the axis, is given. In the fourth column the values are indicated of which the numerals are used to mark the points of the scales.

Axis	horizontal distance	scale function	numerical value
1	0	$20(1 - e^{-\alpha z})$	αz
2	zero line	$20(1+W)/(2.111+W)$	$W = (z_0 - 1)v_c / (z_0 + z_s)k_0$
3	70	$18(1 - e^{-\alpha \psi})$	$\alpha \psi$
4	zero line	$20\alpha z(\alpha z + 1)$	αz
5	zero line	$20(2\alpha z_0 - 1)$	αz_0
6	74.7	$16.8(1 - e^{-\alpha \psi})$	$\alpha \psi$
7	88.5	$13.5(1 - e^{-\alpha \psi})$	$\alpha \psi$
8	99.3	$10.46(1 - e^{-\alpha \psi})$	$\alpha \psi$
9	140	pivot line	immaterial
10	210	pivot line	immaterial
11	280	$500 W$	$W = \frac{\alpha(z_0 - 1)}{\alpha(z_0 + z_s)} \frac{v_c}{k_0}$

A zero line is an oblique axis which links the zero points of the two adjacent parallel axes, a pivot line is an arbitrarily calibrated axis on which the reading lines intersect. The value, which might be read from such an axis is, however, as an intermediate result of calculation, immaterial for the solution.

The axes 7 and 8 are given in addition for the cases when higher values of

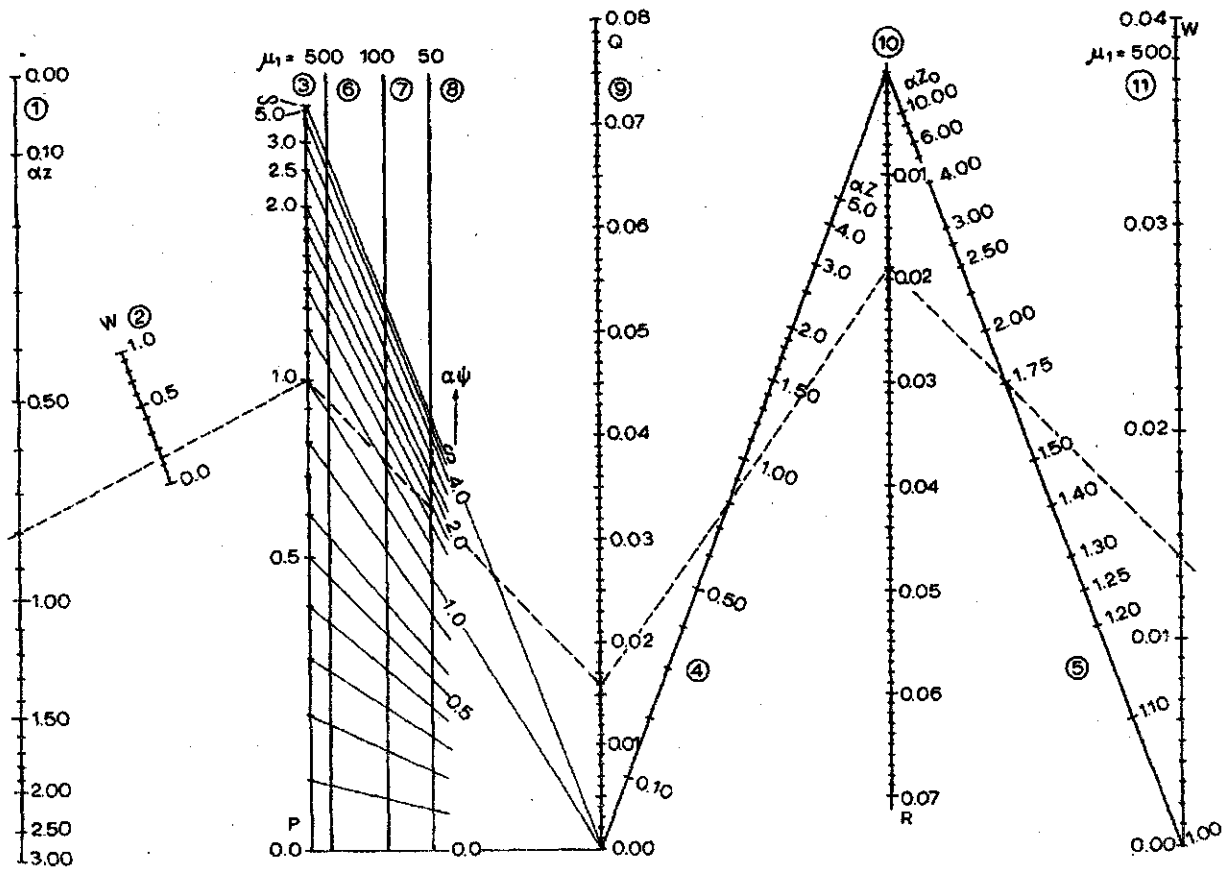


Fig. 21. Nomogram for the calculation of unsaturated upward flow for decreasing extraction at increasing depth. The axes 6, 7 and 8 should be used in case the scale along axis 11 is taken $\mu_1 = 500$, 100 or 50

W have to be used, surpassing the highest values indicated on scale 11. For values of W as indicated on axis 11, the value of $\alpha\psi$ should be read on scale 6. If the values along axis 11 are multiplied by 5 or 10, then the $\alpha\psi$ should be read on scale 7 or 8. The dashed reading lines drawn in the nomogram as an example, are constructed for $W/10$ as appears by comparing the values indicated at the points of intersection on the axes 2 and 11. These values are 0.14 and 0.014. This points at the use of a smaller modulus. Because the nomogram is not used with modulus $\mu = 500$ but with $\mu = 50$, the solution for $\alpha\psi = 1.6$ has to be read from axis 8.

The nomogram can be used for adjustment of the constants α , z_0 and k_0 . The values of z , z_s and $v_c = E_r$ will be available and ψ will be solved from the known soil moisture contents with the aid of the desorption curve. The adjustment is done by assuming a value for α , z_0 and k_0 . If for each observation the reading lines are drawn, using the observed values of z , z_s , v_c and ψ , then with correct values of α and k_0 the lines will all intersect on axis 5 for αz_0 . The value assumed for z_0 to calculate W must compare with the average of the values read at the intersections on axis 5. If these values are not equal, other combinations of α and k_0 have to be tried out till the data fit sufficiently with the observations. This fitting of values to the equation of the soil moisture profile is rather complex, but the results will show that often some of the constants can be neglected. In case of a good pervious soil the value of k_0 is high and this means as was already mentioned, that the formula simplifies to $z \approx \psi$. This simple solution is, however, of restricted use if the equation is intended to help determining at what value of z the capillary rise becomes too small to supply the plant with sufficient water. In this case the aim of the investigation is, to assess to which extent the soil dries out to a moisture content lower than the equilibrium moisture profile. This is shown by the value of ψ , which then is larger than z .

Evaporation and soil moisture content

The moisture needed for the evaporation by plants is derived from the main root zone. The root zone is replenished, if not by rain, then by capillary supply from the subsoil. Between the capillary supply v_c of the previous paragraph and the real evaporation E_r exists a direct relation.

The evaporation, as part of the transport process with respect to the uptake of substances needed for plant growth, is based on the same laws of linear flow and minimum stress as the uptake of plant nutrients and identical basis leads to an equation of similar shape. In this formula the soil properties define together the flow resistance parameter A, which was assessed to read as follows (Visser, 1964*):

*Visser, W. C.: Moisture requirements of crops and rate of moisture depletion of the soil. Inst. for Land and Water Managem. Res. Techn. Bull. 32, 1964.

$$A = \frac{4 L k_o \psi_e^n}{(n-1) d^2 \left\{ \ln \left(\frac{d}{r} \right)^2 + \left(\frac{r}{d} \right)^2 - 1 \right\}}$$

d = radius of moisture extraction
for a single root

r = radius of the root

L = thickness root zone

k_o = capillary conductivity k_c extrapolated to $\psi = 0$

ψ_e = air entry stress

n = exponent in equation $k_c = \left(\frac{\psi_e}{\psi} \right)^n k_s$

k_c = capillary conductivity at moisture stress ψ

Because plant properties as the thickness L of the main root zone, the density of the root system $1/d^2$ or the average root diameter $2r$ are difficult to assess in the soil, the value of A will usually be determined by an adjustment technique and not from direct measurements. The equation may, however, be useful to transfer experience from one set of conditions to another.

The evaporation is calculated from an equation which is similar to the growth equation and which reads as follows:

$$\left(g - \frac{E_r}{E_o} \right) \left(A - \frac{E_r}{v^{\ell} - v_{wp}^{\ell}} \right) = F_3$$

The equation is represented graphically by fig. 22, assuming that the moisture content v_{wp} at wilting point may be neglected, or the omission being compensated by correcting the factor A and the exponent ℓ as was done in the paragraph on methods of curve fitting and assessment of parameters. As value for the potential transpiration, a calculated value of E_o can be used, but often data of an evaporation pan are more practical.

Adjustment of constants

In the adjustment of the constants g , A , ℓ , v_{wp} and F_3 one will have to concentrate on the first three. The importance of the value of F_3 is restricted and by taking $F_3 = 0.05 g A$, not much need for further adjustment will remain. The adjustment is carried out by making use of the property of the equation that two curves, given graphically and relating v to E_r , for two values of E_o can be brought to coincidence by shifting the curves along the oblique asymptote $E_r = A(v^{\ell} - v_{wp}^{\ell})$. In the same way two curves for E_r against E_o for different values of v , can be brought to coincidence by shifting the curves along the $E_r = gE_o$ asymptote.

This shifting is carried out in graphs, obtained by plotting against each other: $\log E_r - (\log \bar{E}_o - \log E_o)$ and $\log v - \frac{(\log \bar{E}_o - \log E_o)}{\ell}$

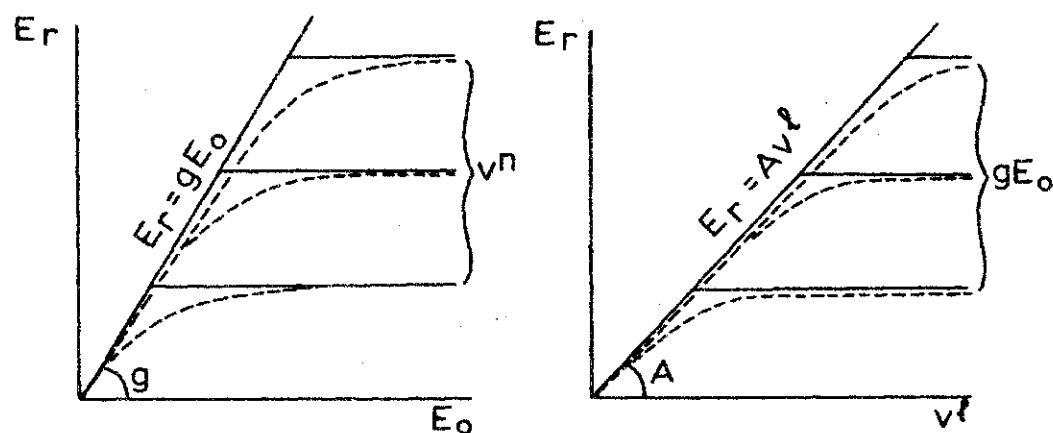


Fig. 22. Shape of the relation between real evaporation E_r , potential evaporation E_o and soil moisture content v according formula 3

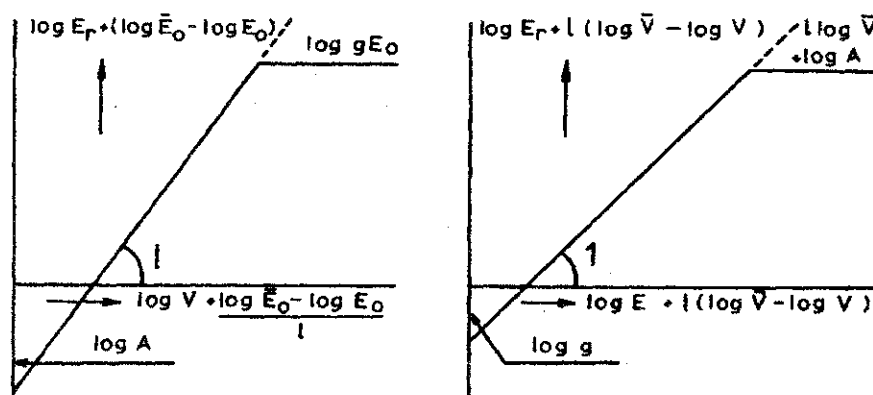


Fig. 23. Adjustment of formula 3 is carried out by an oblique shift, which brings for different levels of v or E_o the curves to coincidence. The shift is represented by the terms $(\log \bar{E}_o - \log E_o)$ and $(\log \bar{v} - \log v)$

as well as:

$$\log E_r - \ell(\overline{\log v} - \log v) \quad \text{and} \quad \log E_o - \ell(\overline{\log v} - \log v)$$

The values of $\overline{E_o}$ and \overline{v} are levels of E_o and v to which the other sets of observations are reduced. They can be freely chosen.

For this shift a first approximation of ℓ has to be made, to be able to calculate the correction terms, but then the value of the tangent ℓ from the asymptote can be used for a second elaboration and the values of $\log g$ and $\log A$ are found as the intercepts on the vertical axis. In fig. 23 the principle of this curve fitting technique is shown.

An elaboration with data of sufficient accuracy will show, that the value of g is not entirely constant, but will vary during the year. This variation depends on the degree to which the crop covers the soil and also in the height of the crop effecting turbulence of the air. These effects will only be of importance if a high accuracy is needed. In this discussion they will be neglected. Another influence which is left out is the influence of variations in moisture content in the root zone. This can be treated by inserting in the equation for the evaporation the flow through n horizontal layers. The term T under consideration then takes the following shape:

$$T = \left(1 - \frac{E_r}{\sum_{i=1}^n A_i (v_i^{\ell} - v_{wp_i}^{\ell})} \right)$$

By inserting this formula in equation 3 the moisture flow equation for a number of parallel directions of flow is given, with conductivity constants A_i and gradients $v_i^{\ell} - v_{wp_i}^{\ell}$ for each layer.

Evaporation and groundwater depth

The relation between soil moisture content and evaporation is a rather direct one, not accessible to disturbing influences. A point of practical interest often is, however, the relation between groundwater depth and evaporation.

This relation can be determined, because the upward flow of moisture through soil, plant and atmosphere can be measured at four successive points of the flow path. Measurements can be carried out in the atmosphere by determining the vertical vapour transport. The measurement in the plant frequently used is the moisture transfer through the stomata. The equation for the moisture movement in the rootzone was discussed in the previous paragraph.

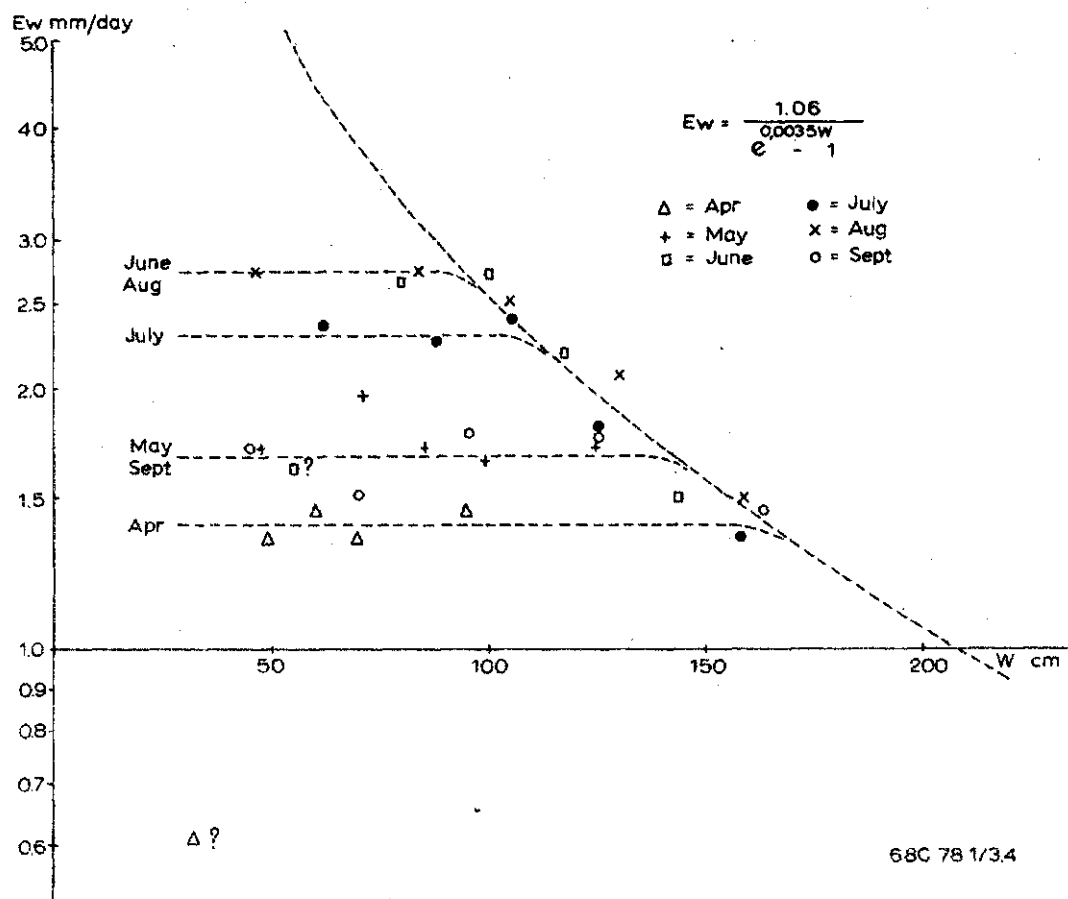


Fig. 24. Example of the adjustment of data on real evaporation with respect to the potential evaporation in case of limiting climatological conditions and to capillary flow in case of limiting soil moisture contents

The measurement of the capillary upward flow can also provide a basis for determination of the real evaporation. In the discussion concerning the soil moisture profile the formula relating groundwater depth to the velocity of upward flow was discussed. This equation only holds in the range of depth, where the soil moisture conditions represent the limiting factor for evaporation. For the range of groundwater depth, which includes the wetter conditions, the following formula holds:

$$\left(g - \frac{E_w}{E_o} \right) \left(k_o - \frac{e^{\alpha z} - 1}{1 - e^{-\alpha(\psi - z)}} E_w \right) = F_3$$

In this formula further simplifications as already were discussed, can be applied. In fig. 24 an example is given for sandy soil under normal farming conditions.

It should be remembered, that this equation only holds for situations of steady upward flow and should therefore not be used for conditions prevailing after rainfall. Because however, in normal agricultural soils the rainwater is quickly evenly divided over the soil profile, the equilibrium conditions are sufficiently restored in one or two days. Therefore the equations can be used for assessing the evaporation if a time-unit of a week or a fortnight is chosen, even if some of the days of the time-interval have been rainy.

Discharge and groundwater depth

Experience has shown, that in areas with an impervious layer at depth D and a wide spacing L of ditches and rivulets, the discharge equation is rendered with sufficient accuracy by:

$$A = \frac{8kd}{L^2} (W - S) = C(W - S)$$

W = groundwater depth

S = water level in ditch

k = conductivity

d = thickness of the pervious layer

L = drain spacing

$$C = \frac{8kd}{L^2}$$

The second power term $(W-S)^2$ of the common drainage equation can be neglected. What however cannot be neglected is, that generally at least two drainage bases can be assessed from the observations; a shallow and a deep

one, the first consisting of furrows or field drains, the second of deeper water courses at larger distances. This shows up in the discharge curve as a more or less sharp bend, which has to be distinguished from the gradual transition from a flatter to a steeper part of the curve in case of a second power relation.

The combination of the effect of drainage bases on different levels can be achieved by multiplication of the terms for each drainage process. In such a formula the drainage equation should be written as a constant minus the ratio of flow to gradient, or as $C - D/(W-S)$. The product of two or more of such terms has to be constant. This is the same procedure as used for the yield equation and the equation for evaporation.

The formula becomes:

$$\left(\frac{8k_1 d_1}{L_1^2} - \frac{D}{W - S_1} \right) \left(\frac{8k_2 d_2}{L_2^2} - \frac{D}{W - S_2} \right) = F_2$$

and more of these terms of the same construction can be inserted if more drainage bases show up. For instance a subsurface flow to rivers at a considerable distance may be present, for which the gradient ΔW is constant. The thickness of the layer of flow d then depends on the upper limit of this layer and it varies with the depth of the groundwater W . The thickness of the layer is now inserted in the formula as $d-W$.

Such a term T , added to the previous equation, would read:

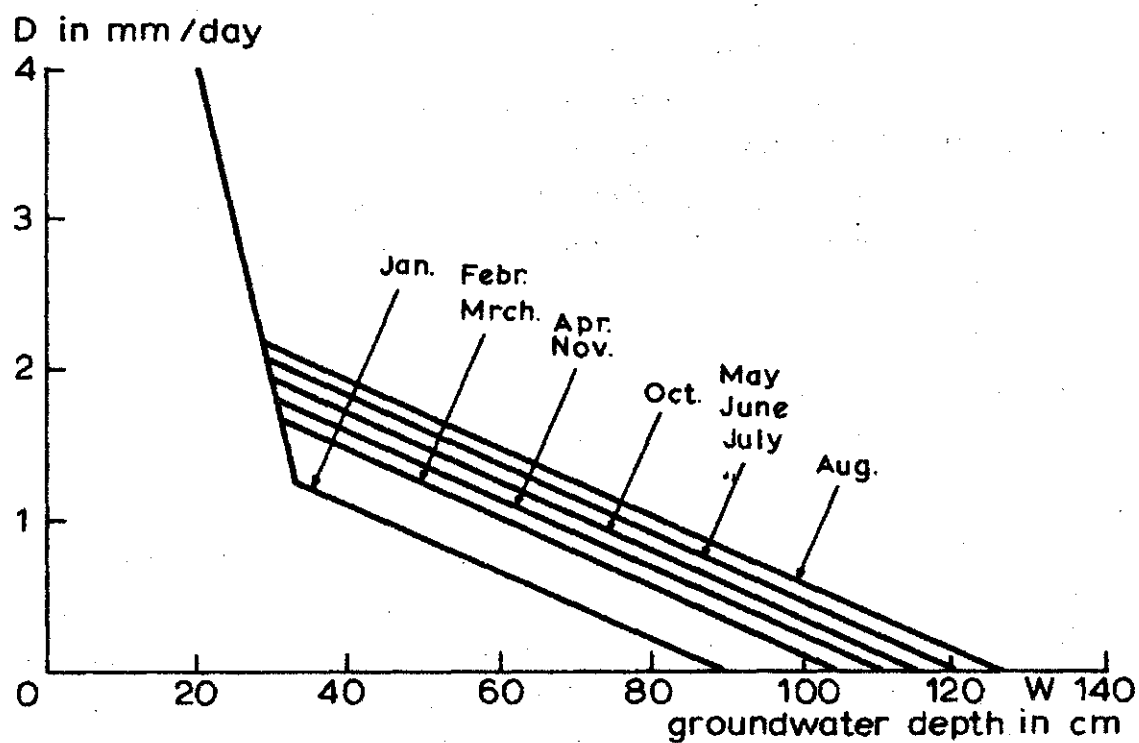
$$T = \left(k(d-W) - \frac{D}{\Delta W} \right)$$

The value of F_2 determines whether the transition from one branch of the discharge curve to the other is an abrupt or a gradual one. An example of such a discharge relation is given in fig. 25. This example describes a case in which the drainage base - a distant river - varies in level as a result of variations in run-off.

The adjustment of the data obtained for instance from an analysis of the groundwater measurements, is carried out by plotting A and W or A and S for different classes of S resp. W against each other and drawing straight lines through the scatter diagram.

Use of the formulae

The formulae may strike many designers of projects as rather complicated. The aim has been, however, to choose the simplest equations which



$$\left\{ 1 - \frac{D}{\frac{8k_1 d_1}{L_1^2} (W - a_1 H - b_1)} \right\} \left\{ 1 - \frac{D}{\frac{8k_2 d_2}{L_2^2} (W - a_2 H - b_2)} \right\} = F$$

$$a_1 H + b_1 = S_1 \quad a_2 H + b_2 = S_2$$

Fig. 25. In the water balance often the influence of two drainage bases is indicated, of which the lower one shows the effect of fluctuating water levels in the main water courses

can be applied over the largest range of hydrologic, climatic and agricultural conditions. And compared with the equations to which specialist studies often lead, the unwieldiness may probably be considered to be not too excessive.

The equations make it possible to use hydrologic soil properties to explain actual water balance situations. Also the response of crops or natural vegetation to moisture conditions can be predicted. One of the valuable points is, that the formulae allow to eliminate uninteresting variables and show that any magnitude can be expressed in each of the others. This may clarify relations which remain generally vague if one has not chosen them as object of basic study.

Groundwater levels are easy to determine and have a practical significance. Soil moisture stresses are an instance of a magnitude which is of restricted practical importance and is far less easy to determine. Still for scientific reasoning, the soil moisture stress is normally used and this makes the results, expressed as function of such a variable, so difficult to apply in practice. The equations obtained show how theoretical problems as evaporation, which are normally expressed as functions of ψ , now can be represented as functions of the groundwater depth. In the case of evaporation for instance the following result is obtained:

$$\sqrt{\ell \frac{E_r (gE_o - E_r)}{(gA - F_3) E_o - AE_r}} = \frac{-1}{\alpha} \ln \left\{ e^{-\alpha z_s} \left(\frac{\alpha z_o - 1}{\alpha(z_o + z_s)} \cdot \frac{E_r}{k_o} + 1 \right) - \left(\frac{\alpha(z_s + z_o - 1)}{\alpha(z_o + z_s)} \cdot \frac{E_r}{k_o} \right) \right\}$$

This relation is obtained by taking the real evaporation E_r equal to the rate of capillary supply v_c at the soil surface, where $z = z_s$. The equation shows that a direct relation exists between E_r and z_s , defined by a number of soil and crop constants. Included in the formula is E_o as the only variable independent of the groundwater depth z_s and it can therefore not be eliminated. In the same way the storage capacity can be expressed as a function of z_s , whilst the discharge is already given as a function of this technically important magnitude of the groundwater depth.

This possibility of expressing the water management problems as a function of z_s can be used in two ways. It can be used to study the influence of climatological conditions on the water balance for different drainage and irrigation situations. But it also can be used to predict the effect of these

measures on crop production.

Prediction of water balance variations

If the correct constants in the formulae are found by some adjustment process, it becomes possible to calculate for each of the values of the water balance terms D , E_r and $\mu\Delta W^*$ with $\Delta W^* = 1$, what groundwater level matches with the intensity of each of the water balance terms. For ΔW , a unit value is taken to calculate what amount of rainfall provides, next to evaporation and discharge, an increase in storage of 1 cm. It is possible, by using the formulae, to calibrate four axes with the mm-value - using the same scale units for the various axes - at one side of each axis and - except for the rainfall axis - with the corresponding value of z_g at the other side. The result is shown in fig. 26.

The four axes are united into a rectangular square, along which the direction of increasing mm is going clockwise, save for the axis for moisture storage, of which the direction of increase is anti-clockwise. This nomogram is read by a rectangular cross on transparent paper. If discharge and evaporation are marked on opposite axes, one leg of the cross is laid through two points of these two axes with the same value of z_g . Then the cross is shifted, with the first leg still intersecting at the same z_g -values on the A - and E_r -axes, till the second leg intersects the $\mu\Delta W^*$ -axis at the same value of z_g . The mm of rainfall at the point of intersection with the fourth axis is read. This is the amount of rain R^* which would account for the discharge and evaporation obtained with this groundwater level and the change of storage of $\Delta W = 1$ cm.

The difference between the real rainfall R and this calculated rainfall R^* represents the actual rise or lowering ΔW of the groundwater level according to the relation $\Delta W = \frac{R-R^*}{\mu} + 1$, and by adding this value of ΔW to z_g , the groundwater table at the beginning of the second time interval is produced.

The nomogram has to be constructed for different values of the ditch water levels S of the discharge formula and for different values of E_0 for the evaporation and storage formulae. This can be done by constructing cartesian side nomograms with the common rectangular axes, which allow to find for each S or E_0 the values in mm for D , E_w and $\mu\Delta W$ which match with a certain value of z_g .

The nomogram enables one to calculate for an initial value of z_g what

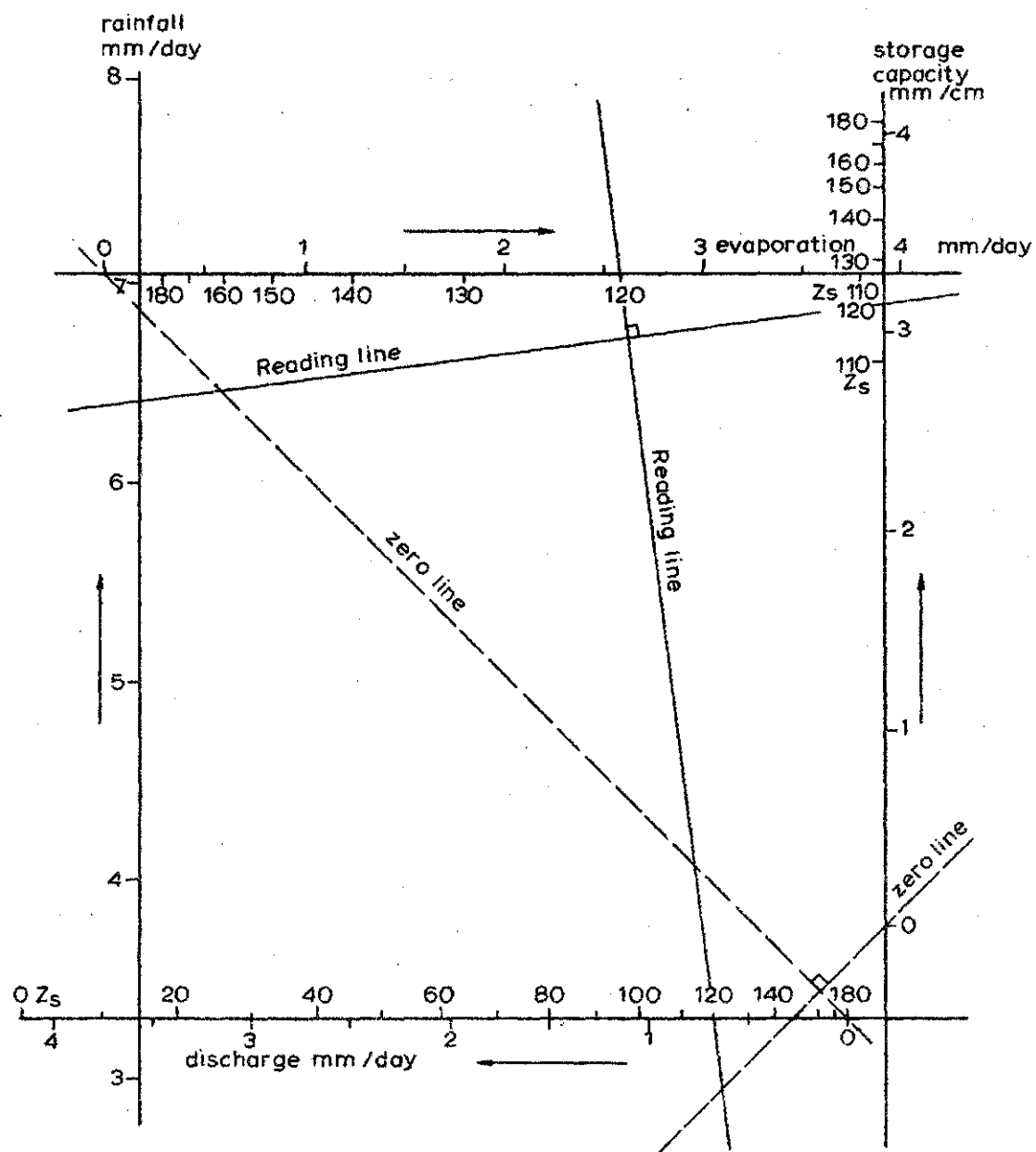


Fig. 26. Nomogram for the calculation of the response of the water balance terms in the course of time to the daily or weekly rainfall by assessment of ΔW

the change ΔW_1 is for a given rainfall, then to calculate the groundwater depth $z_{s2} = z_{s1} + \Delta W_1$ for the next interval and then again to assess the value of ΔW_2 . This can be carried on for a sequence of climatical conditions, for instance a critical period, in order to find out what the moisture conditions will be before and after improvement of the drainage system, or for different moments and quantities of supply of irrigation water. Any alternative solution can be worked out and any strategy of using the management techniques, taken into consideration for actual construction can be sorted out in advance.

Prediction of crop response

The prediction of crop response depends largely on the ability to determine the coefficients for the successive stages of plant growth. Situations of excess or deficiencies of water which seldom occur, often are of great importance for the assessment of the profitability of a project. If data for periods with excessive rainfall or prolonged drought are not available during the short time of preparation of a project, this will invalidate the resulting design. Therefore experiments and observations on crop growth should preferably be started as soon as possible and be carried out as long as possible.

An example of such an elaboration of data is given in fig. 27, in which, in order to simplify the problem, the discharge is left out.

The four remaining formulae and the constants which were used, are given. The formula for the moisture profile is used for $z_0 = \infty$, the lysimeter formula for the case that the water which evaporates is directly replenished by subsoil infiltration. The moisture profile is assumed to be a two layer case, the root zone having a constant moisture profile and the subsoil a moisture distribution according the formulae 2 and 4 mentioned in fig. 27. As common variable the moisture content in the root zone is taken for a change. It could have been the groundwater depth as well.

The arrow and number pointing at each curve give the formula which was used to calculate that curve. The depth below the soil surface, where the curves (2+4) for the soil moisture profile intersect the axis at the right hand, shows what groundwater depth z_g goes with each rate of evaporation E_w .

The lower half of the graph also shows, what relation exists between the moisture content in the root zone, the real evaporation and the groundwater depth. The upper half shows what the effect of these magnitudes is on crop production. The graph and the formulae deal as well with low moisture contents and water deficiency as with high moisture contents and water logging.

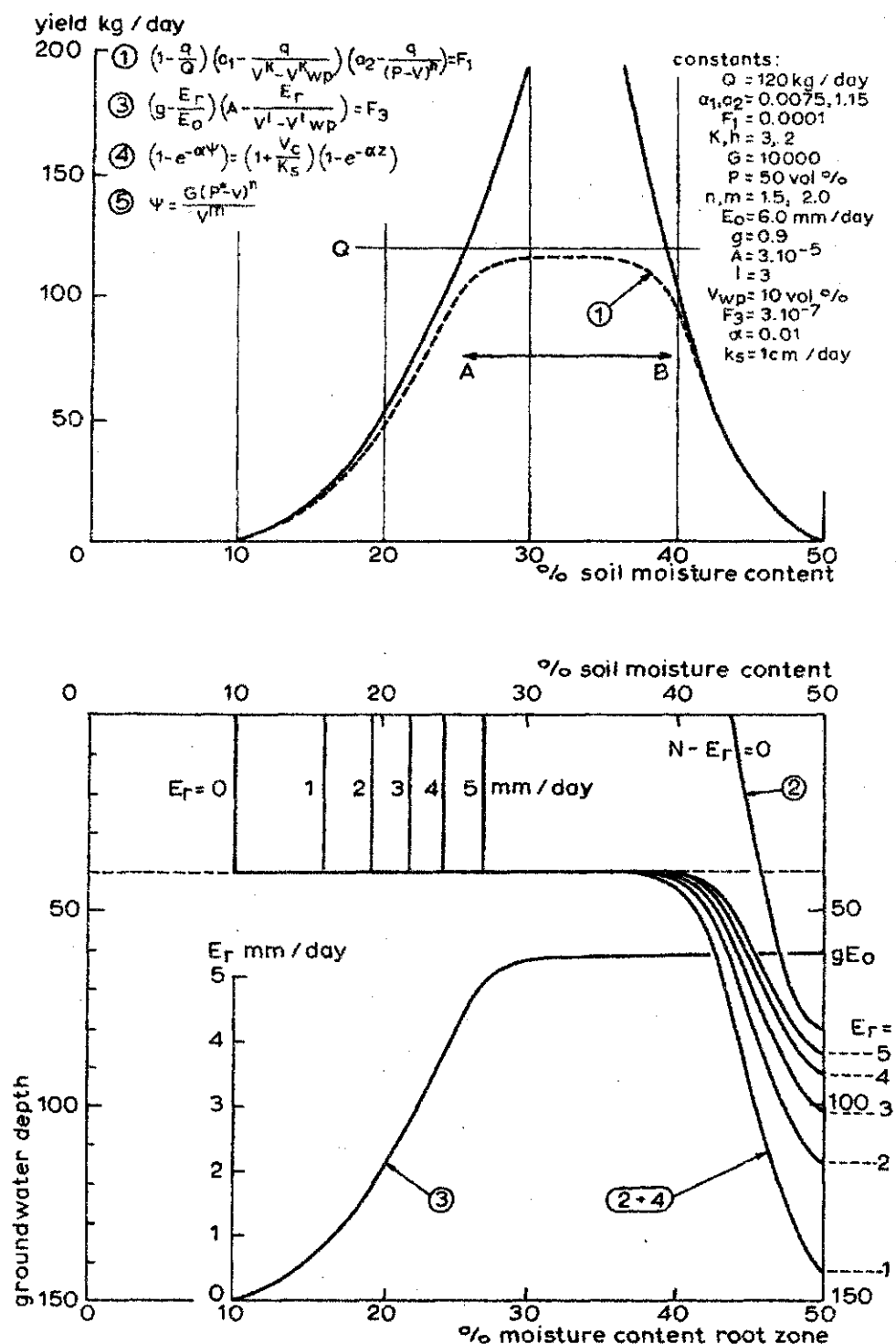


Fig. 27. Example of the response of the crop yield on groundwater depth, moisture content in the root zone, potential evaporation and capillary rise according the formulae 1, 3, 4 and 5

The intermediate range A-B of moisture conditions with optimal growth in fig. 27 is clearly the range at which the project should aim. The lower the maximum attainable yield Q, the wider the optimal range and the easier the design and management of the project. In areas with low production, a very crude assessment of the moisture relations will do. A high production requires a very exact handling of the water management measures on penalty of transgressing the narrow limits of maximum production, and stray off into the ranges of suboptimal growth.

Assessment of drain depth

The most accurate way to predict the required drain spacing and depth is the one, discussed in relation to fig. 26. In the calculation of the variations in the water balance the elaboration can be carried out with a number of combinations for drain spacing and depth. For each combination, the sequence of groundwater depths can be assessed. By noting at the same time the moisture or air content on the μAW^* -axis, the yield curve of a type as depicted in fig. 27, if calculated with the proper constants, can be used to predict what yield depreciation will occur for each combination of drainage constants. This method is, however, the most laborious.

The depth of drainage depends on the point where in fig. 27 the flat summit changes over in the slopes of the yield curve. These points of transition can be recalculated from moisture percentages into groundwater depths. The depth of drainage has to be taken equal to this groundwater depth, calculated from the moisture content, to which is added the average head of pressure between the middle of the field and the drain.

If the average runoff is equal to D_a , then the average head of pressure W_a is calculated as

$$D_a = \frac{aW_a + bW_a^2}{L^2}$$

depth

The highest permissible drainage W_n then is the maximum groundwater level W_m according to the yield curve, plus this pressure head W_a required for draining away the rainfall surplus. The upper limit for the depth of drainage W_n is equal to

$$W_n = W_m + W_a$$

The lower limit depends on the lowest water level, acceptable according to the yield curve, for which fig. 27 gives a moisture content of the order of 25%, an evaporation of 4.5 mm per day and a groundwater depth at the right hand side axis of the lower graph of 90 cm.

The upper level will require some rigid definition of what is intended with the drainage. If it is only spring drainage, then a certain air content in the root zone has to be indicated that should be present in order to start spring growth. The maximum growth rate ^{in spring} may be of the order of 25 kg per day, which goes, as depicted in fig. 27 and formula 1, with an air content of 5%. This, according to formula 2, means a groundwater depth of about 50 cm. If an air content had been required of 10%, then a groundwater depth of 180 cm would have been calculated.

Would one centre the problem of drainage on the discharge of summer rainfall, then the required air content for unhampered growth at a high level of production would have been taken and an evaporation intensity would have to be defined which - together with the groundwater depth - governs the air content in the root zone.

If the evaporation is taken 5 mm and the required air content 10%, then in fig. 27 in the lower graph the value of the water depth on the right hand side axis is read to be 70 cm. The distribution of rainfall over the year and the response of the crop to the moisture relations will determine, what the depth of drainage should be, but it will be clear that for each year a slightly different value will be the best one. And during the year the results of the calculated optimum depth of drainage neither will be constant. The depth of drainage, due to its basic meaning, is an average value of wide variations, each of which has only a momentary validity.

Assessment of the drain spacing

The drain spacing follows from the depth of drainage and the agricultural requirements. The actual spacing should be calculated by inserting rainfall data in the water balance formula, assessing the resulting water table height for a given drainage depth and spacing, and calculating the yield response with the productivity function. If this is considered to be too laborious, then this calculation is replaced by an elaboration which uses the rainfall frequency, a storage volume and a linear function for the integrated discharge. This latter assumption means that the discharge is constant and not dependent on the amount of drainable moisture, which is unsatisfactory, however. Therefore three acceptable formulae are suggested as a basis for this simplified solution for the drain spacing.

These formulae are:

$$\text{a) } z = \frac{G\mu^n}{(P-\mu)^m} \quad \text{b) } -\frac{dS}{dt} = \frac{az + bz^2}{L^2} \quad \text{c) } \Sigma R = pt^q$$

Equation a) is the desorption formula. If the value of μ does not vary too much, the denominator can be taken constant. Further it is known that the exponent n is often of the order of 1.0.

Formula b) is a drainage equation in which S is the volume of water that in the course of time can drain away. The formula especially is used as a function between L and z .

Equation c) gives for a certain probability K of transgression the rainfall sum as a function of the length of the time interval for which this rainfall sum and recurrence frequency are calculated. The exponent q and the constant p have for the shorter lengths of the time interval for a certain rainfall station a value as follows from fig. 28.

The first formula will be used in its simplified form:

$$z = G^*\mu \quad \text{or} \quad \mu = \frac{dS}{dz} = \gamma z$$

From the latter formula follows:
$$\text{d) } S = \frac{\gamma}{2} z^2$$

In formula c) the value of ΣR is taken equal to S , meaning that the rainfall sum after t days with the given probability which determines p and t will just saturate the soil, bring the amount of drainable moisture to its maximum and build up the groundwater table to the soil surface. This is defined by the equations:

$$S = pt^q \quad \frac{\gamma z_s^2}{2} = pt^q \quad \frac{dS}{dt} = \frac{pt^{q-1}}{q}$$

In the latter formula z_s defines the water level at the surface with respect to the drains and represents therefore the depth of drainage.

Now several ways of calculation of the required drain spacing are open. It is possible to equate:

$$-\frac{dS}{dt} = \frac{az_s + bz_s^2}{L^2} \quad \text{and} \quad \frac{dS}{dt} = \frac{pt^{q-1}}{q} \quad (\text{g})$$

In this case a drain spacing is calculated, which gives a runoff at the moment that the water table just has risen to the soil surface, equal to the intensity of the rainfall at the t^{th} day according to the probability function. This means that $\Sigma R = S_{\text{max}}$ and that the increase in ΣR per unit time dS/dt at that moment just can be drained away.

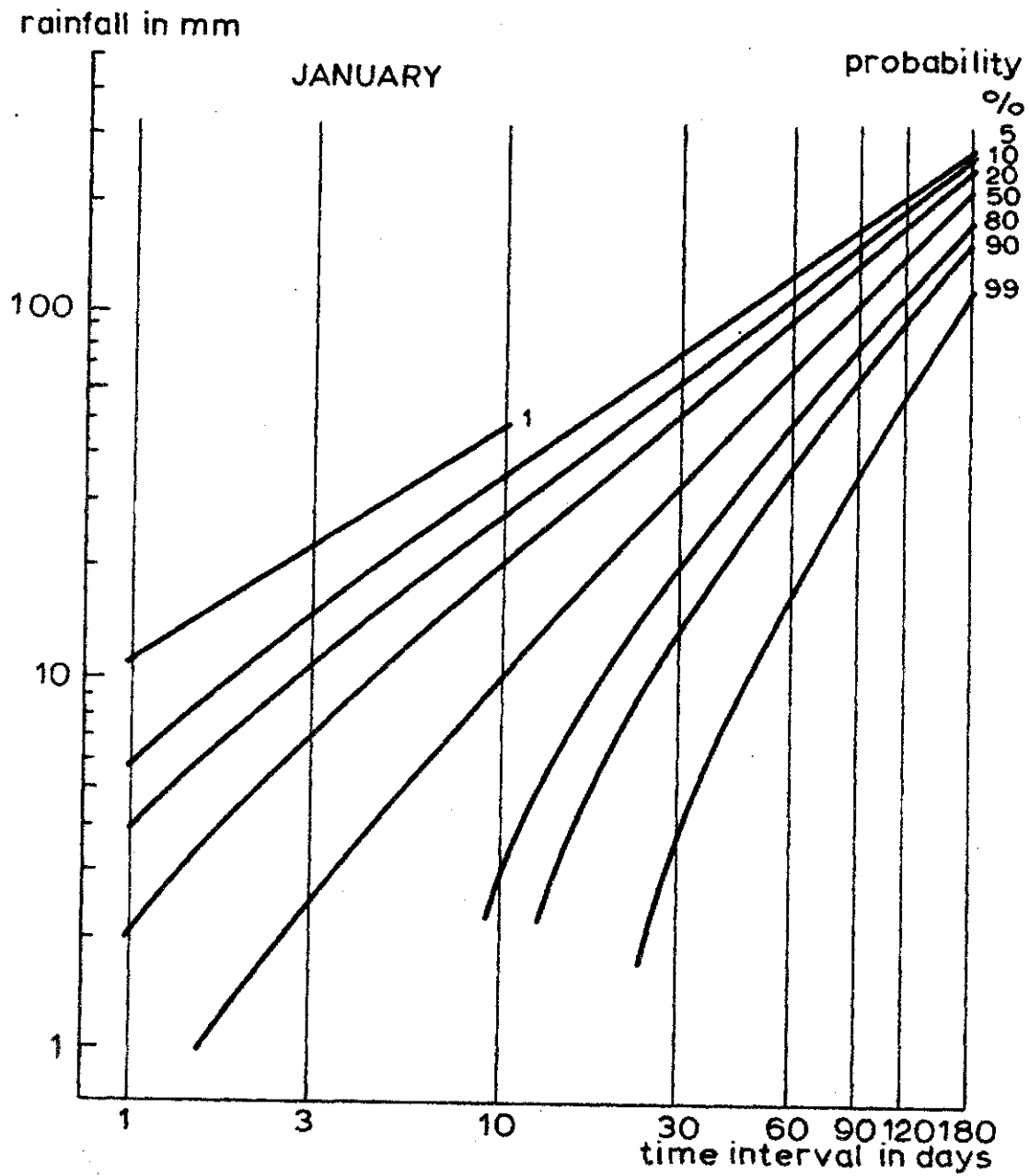


Fig. 28. Example of the rainfall probability in spring in its relation to the length of the time interval

This means that for $z = z_s$

$$L^2 = q \left(\frac{az_s + bz_s^2}{pt^{q-1}} \right) \quad (e)$$

But an other solution is also possible. For this solution it is assumed that a period with average rain intensity \bar{R} is succeeded by a heavy rainfall ΣR over t days. For reasons of easy mathematical solution it is assumed that the quantity ΣR of rain has fallen at the beginning of the t days. In the preceding period the \bar{R} rainfall has built up a water level z_a above the drains and a matching amount of drainable water S_a . The t -day period adds an amount S_r of drainable moisture and a rise of the water level to z_s . Now the requirement is made, that in the same t days the drainage system is able to drain away the amount of rainfall ΣR . The probability of recurrence of transgressions of this requirement - at which the water rises above the soil surface or the water cannot be drained away in the same t -days - is the dependent variable.

The latter solution requires the integration of the discharge, which is obtained as follows.

From formulae b) and d) the discharge sum can be deducted:

$$- L^2 \frac{dS}{dt} = az + bz^2 \quad \text{or} \quad - L^2 d\left(\frac{\gamma z^2}{2}\right) = \alpha z + bz^2$$

$$- \gamma L^2 z \frac{dz}{dt} = az + bz^2 \quad - \frac{\gamma L^2}{b} \frac{dbz}{a + bz} = dt$$

From the formulae follows:

$$\frac{0.4343 b}{\gamma L^2} (t_2 - t_1) = \log \left(\frac{a + bz_1}{a + bz_2} \right) \quad (f)$$

With the formulae b), c), d) and f) the design requirements can be solved. As an example of the formulation of the requirements the following problem is raised:

A field under the influence of the average winter rain will have a discharge of $\bar{R} = 3$ mm, a groundwater level $z_a = 70$ cm and a matching amount of drainable water S_a . Then in a time interval of $t = 5$ days and a probability K of 0.5%, an amount of moisture S_r , which can be calculated from these latter values, is added to the drainable moisture and a groundwater level z_s will occur. A further requirement is that the rainfall sum $\Sigma R = S_r$ has to be drained away in the same time interval $t = 5$, so that after t days the original equilibrium is restored, at which with the original water level z_a

a discharge of $\bar{R} = 3$ mm is obtained. Finally the time of $t = 5$ days has to indicate an optimum. An interval $t < 5$ will mean a water level that has not yet reached the soil surface. An interval $t > 5$ will indicate that the water level recedes from the soil surface because the drainage capacity exceeds the increase of the rainfall sum for an incremental increase in time.

As will be demonstrated, the drainage requirements mentioned are not simultaneously achievable. More requirements are aimed at than the number which will give a full solution. It is of importance, that one is conscious that often the practical requirements are converging on a generally desired goal, but that not all the requirements are converging on the same goal. One wishes more than is possible. An example will be worked out which shows the effect of these conflicting requirements and which at the same time gives an instance of how a solution is calculated.

Example.

The following values for the parameters are taken:

In formula b) $a = 10$ $b = 0.1$ which are valid for dS/dt in mm and L in m.

In formula c) $p = 14.3$ $q = 0.57$ for t in days, R in mm and the probability $K = 0.5\%$.

In formula d) $\gamma/2 = 0.02$

The general parameters have values: $R = 3$ mm $t = 5$ days

$$z_s = 70 \text{ cm} \quad R = S_s - S_a$$

The available formulae allow to calculate the drain spacing in three different ways.

According formula f):

$$L^2 = \frac{0.4343 bt}{\gamma \log \frac{1+b/az_s}{1+b/az_a}} = \frac{0.4343 \cdot 0.1 \cdot 5}{0.04 \log \frac{1+0.01 \times 70}{1+0.01 z_a}} \quad (1)$$

According to formulae b) and g):

$$L^2 = \frac{az_s + bz_s^2}{pt^q} \times qt = \frac{10 \times 70 + 0.1 \times 4900}{14.3 \times 5^{0.57}} \times 0.57 \times 5 \quad (2)$$

According to formulae b), c) and d):

$$L^2 = \frac{az_a + bz_a^2}{\bar{R}} \quad z_a^2 = z_s^2 - \frac{2pt^q}{\gamma}$$

$$z_a^2 = 4900 - \frac{2 \cdot 14.3 \cdot 5^{0.57}}{0.04} = 55.8^2 \quad L^2 = \frac{10 \cdot 55.8 + 0.1 \cdot 3115}{3} \quad (3)$$

The result according these three equations is:

$$1) \ L = 12.02 \text{ m} \quad 2) \ L = 9.75 \text{ m} \quad 3) \ L = 17.02 \text{ m}$$

By varying the probability of recurrence K or the storage capacity χ , an identical solution of the 3 equations for L can be obtained, which proves that one requirement was mentioned above the number which allows a solution. For a value of $\chi = 0.0164$ for instance a solution for $L = 9.75 \text{ m}$ is obtained. A smaller probability of surpassing the rainfall sum $\sum R$ would also provide an identical solution of the three formulae for L at more than twice the rainfall sum used in this calculation and would produce for L the value of 12.02 m .

As was already stated, the drainage requirements which were mentioned are overdefining the problem. The calculation can only lead to a clear result if one knows exactly what one aims at. A disadvantage of the solution is, that the assumption has to be made that the rainfall $\sum R$ is concentrated at the beginning of the time interval t . It means that the solution becomes less applicable the longer the time interval is. It is, however, not difficult to write down the differential equation for an even distribution of the rainfall $\sum R$ over the interval t . The necessary mathematical treatment of such a formula makes this solution uninteresting however. A solution by trial and error would in that case probably be the most advisable.

Summary

The knowledge about water management relations has grown in the latter years to such an extent, that the limiting factor is no longer situated in the field of analyses. The accuracy of the design and the remunerativity of the project becomes more and more linked up with the accuracy of the constants used and the way in which the available knowledge is integrated. If comprehensive planning is advocated strongly these days, it is seldom realized that this means optimizing complicated functions with scores of unknowns, it also means to get the better of a propagation of errors which requires a careful adjustment of observations to available knowledge.

This assessment of the correct constants is, as is commonly known in mathematical circles, a formidable job. It will not be solved by neglecting the difficulties of a correct design, or the deterioration of costs - benefit ratio rising from the omittance of the specialized knowledge needed to solve just these complicated problems.

It is neither an advantage to study the details of the integrated problem along lines of scientific technique without considering how to combine the results into a practical solution, or to seek solutions with a vast amount of constants which may be difficult to derive on the construction site. It also is of slight avail to further the knowledge of one aspect of an integrated problem and neglect the large errors or lack of details in other aspects. The accuracy of the ultimate result depends on the largest error in the chain of relations.

Finally the computer is often presented as the solution of all difficulties. It should, however, not be forgotten that, besides the fact that these instruments are not generally available, a computer requires correct formulae and constants. And in case simplifications are applied, it is more difficult to control what is going on inside the computer, than to check deviations during a non-mechanized elaboration.

From these considerations follows, that it still is useful to select formulae which constitute a compromise with respect to wieldiness, accuracy and transferability. There is a strong need for simplified methods to adjust constants. Further ways of integrating the results of scientific reasoning should be devised, which lead to methods which are able to deal with practical - often multivariate - units of actual project construction. There are not yet many projects carried out in which the computer is used

along lines resulting from physical and plant physiological reasoning. It should be remembered that the linear relations which are often used in computer programs do not preclude that the trees grow into heaven or that benefits decrease below the costs.

The main point of concern with respect to the transfer of the water management experience is, however, that so little use is made of the vast body of knowledge available during the design period or on the construction site. This gap will have to be bridged from both sides, scientific as well as practical.

In this article, the principal idea was to construct a method of solving water management problems along lines which can be followed by anybody who has only the simplest means available of soil moisture research, of yield determination and of mathematical elaboration. Much attention was therefore given to curve fitting techniques, to adjustment of constants and to methods of graphical solution. It is thought to be of considerable importance that a project designer is able to select the best value of a constant from a number of more or less reliable observations. He also should be able to design his own nomograms, in order to get an insight in complicated relations. He finally should master methods to calculate what influence a small variation of an independent variable has on the dependent variable and to check the accuracy of the relations by considering the credibility of the result at extreme values, which often allow the best comparison with practical experience.

The paper aims at showing how formulae can be joined to obtain an integrated solution. The elaboration of alternative solutions is done by varying the values of the parameters of those properties which are accessible to change by technical means. The constants of the drainage formula constitute an example. They enable one to insert a continuous infinite sequence of alternatives in the solution. These alternatives can be expressed in units of discharge, of evaporation, storage, groundwater depth or of plant production as desired and can give a sufficient basis for the assessment of the optimum desirability of any special case in the continuum of differing project designs.

The solution can be shaped to deal with more details by adding other formulae for further aspects. The considerations with respect to accuracy warn, however, that refinement of some details loses its importance as soon as the overall accuracy is limited by other details, known or unknown.